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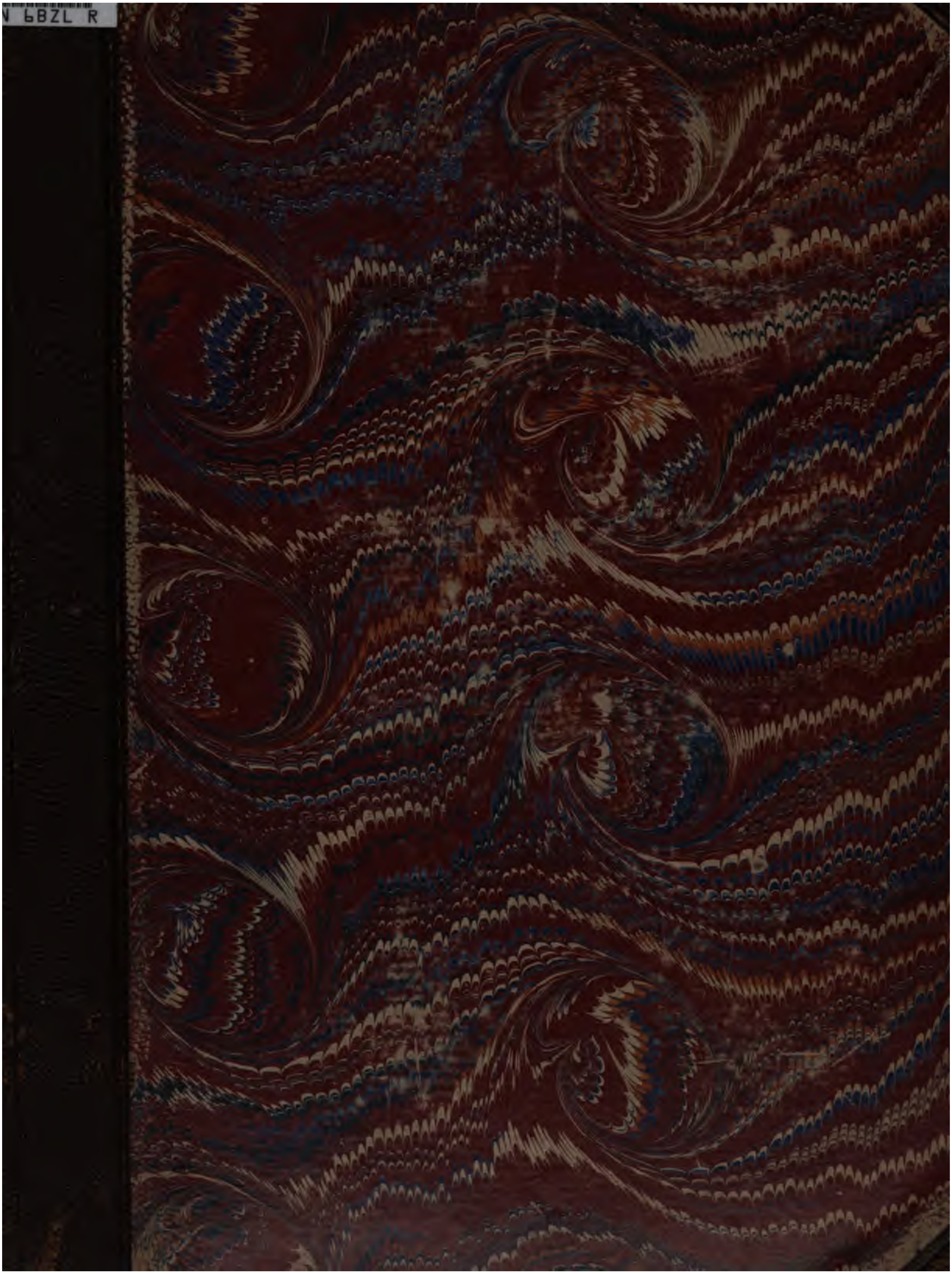

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TO WHICH IS ADDED

A COMPLETE TREATISE ON MATHEMATICAL INSTRUMENTS. ALSO, MENSURATION, TABLES OF THE WEIGHTS AND COHESIVE STRENGTH OF THE SEVERAL MATERIALS USED IN THE CONSTRUCTION OF BUILDINGS, &c.

BY LUCIUS D. GOULD, ARCHITECT, NEWARK.

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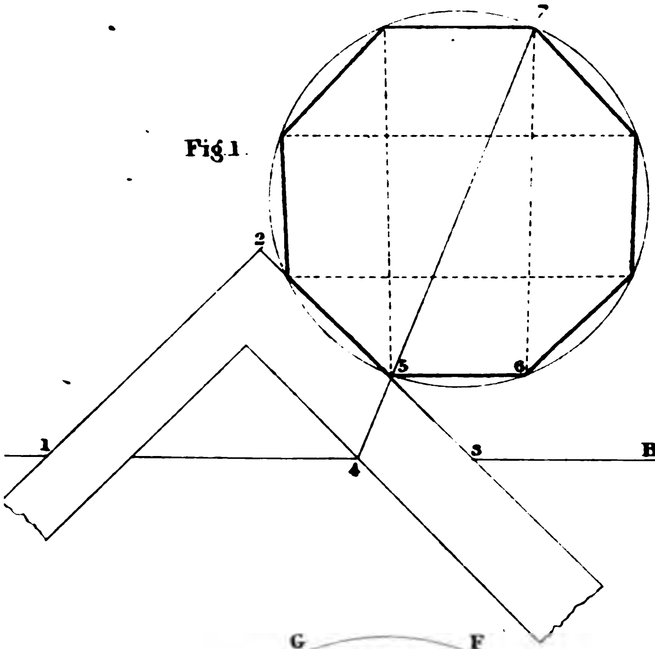
Plate 5

Fig. 1.



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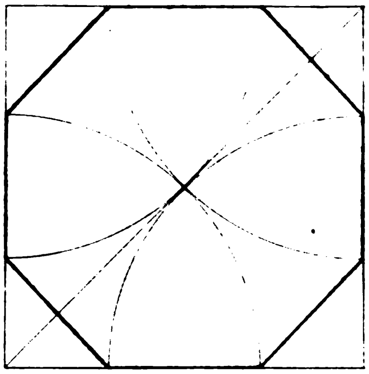

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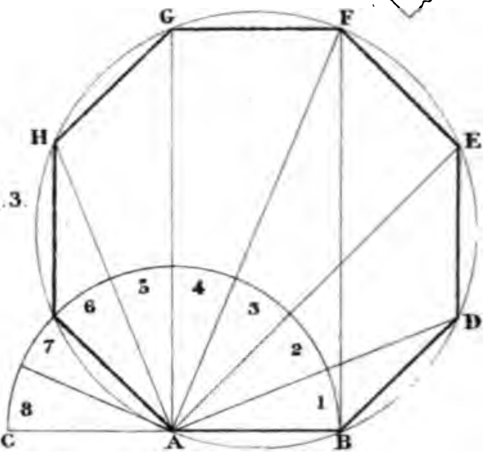

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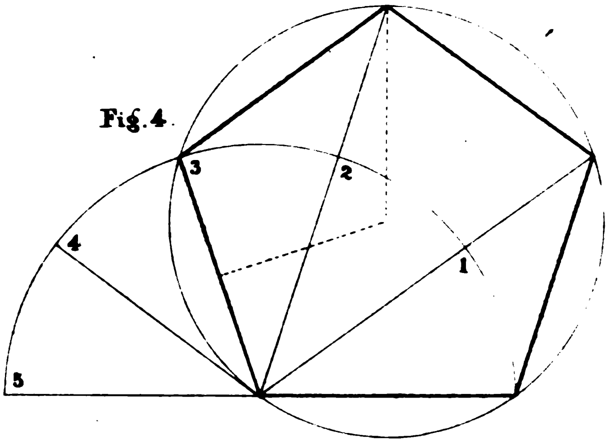

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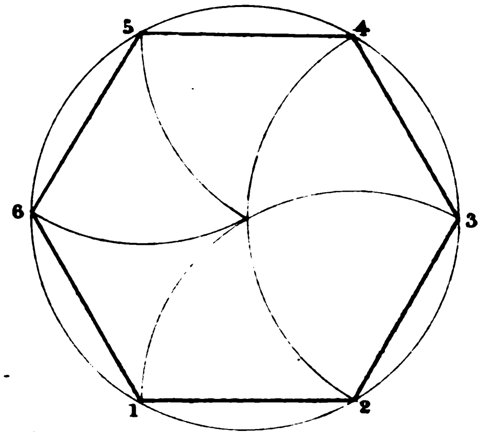

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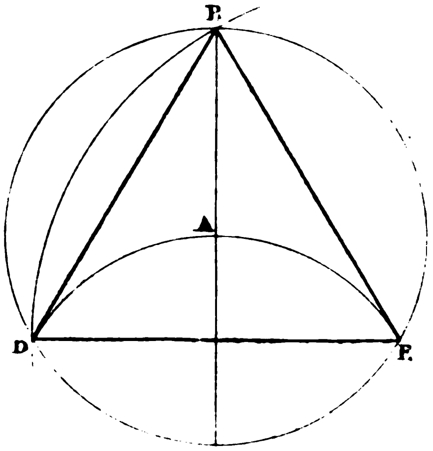


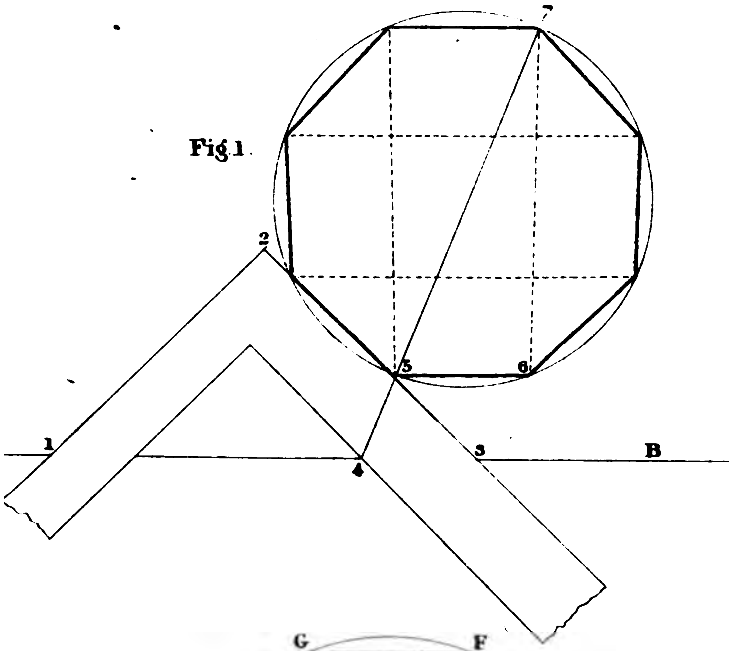
D. M^c Lellan Lith. 26 Spruce St. N Y

required task. It is rather for the benefit of such, than for the experienced workman, that this volume is designed; and should it be the means of promoting their interest, or inciting them to a study of the noble science upon which these arts are founded, the author will feel well compensated for his labor.

It is but due to acknowledge, that the valuable works of Mr. PETER NICHOLSON and THOMAS TREDGOLD, have been freely consulted in the preparation of this volume.

NEWARK, N. J., *May*, 1858.

Fig. 1.



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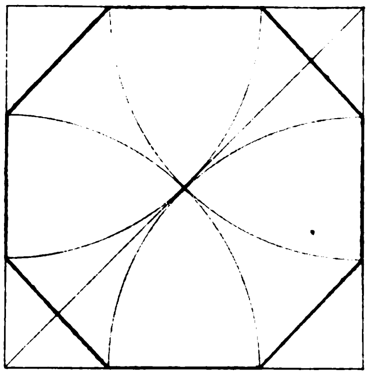

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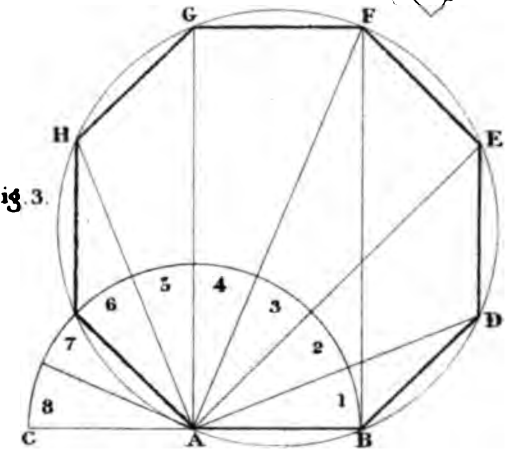

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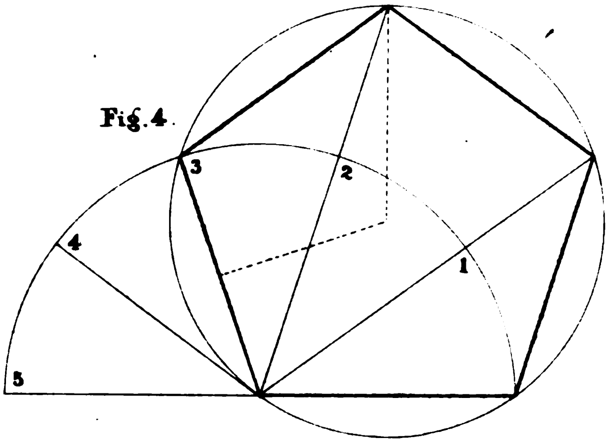

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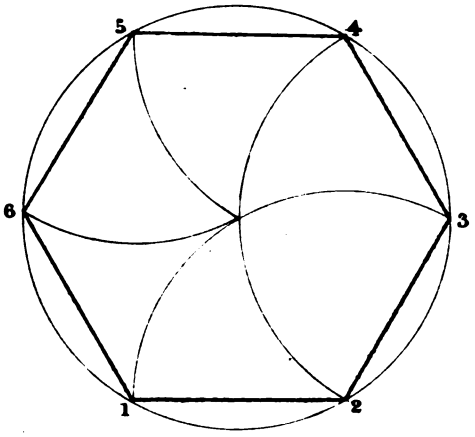
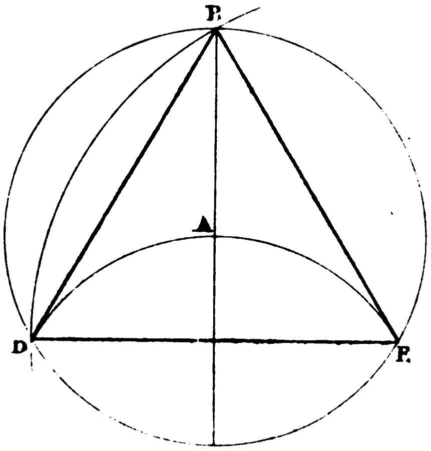


Fig. 6.



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14. Angles are right or oblique; acute or obtuse.

15. A right angle is that which is made by one line perpendicular to another. When the angles on each side are equal to one another, they are right angles, Fig. 11.

16. An oblique angle, is that which is made by two oblique lines, and is either less or greater than a right angle, Fig. 12.

17. An acute angle is less than a right angle, Fig. 12.

18. An obtuse angle is greater than a right angle, Fig. 13.

19. A figure of three sides and angles is called a triangle, and it receives particular denominations from the relations of its sides and angles.

20. An equilateral triangle is that whose three sides are all equal, Fig. 14.

21. An isosceles triangle is that which has two sides equal, Fig. 15.

22. A scalene triangle is that whose three sides are all unequal, Fig. 16.

23. A right angled triangle is that which has one right angle, Fig. 17.

24. Other triangles are oblique angled, and are either obtuse or acute.

25. An obtuse angled triangle has one obtuse angle, Fig. 17.

26. An acute angled triangle has all its three angles acute, Fig. 18.

27. A figure of four sides and angles is called a quadrangle or quadrilateral.

28. A parallelogram is a quadrilateral which has both its opposite sides parallel, and takes the following names, viz.: Rectangle, Square, Rhombus, Rhomboid.

29. A rectangle is a parallelogram having a right angle, Fig. 19.

30. A square is an equilateral rectangle having its length and breadth equal, Fig. 20.

31. A rhomboid is an oblique angled parallelogram, Fig. 21.

32. A rhombus is an equilateral rhomboid, having all its sides equal, but its angles oblique, Fig. 22.

33. A trapezium is a quadrilateral which has not its opposite sides parallel, Fig. 22

PLATE II.

34. A trapezoid has only two of its opposite sides parallel, Fig. 1.

35. A diagonal is a line joining any two opposite angles of a quadrilateral, Fig. 2

36. Plane figures that have more than four sides are generally called polygons and they receive other particular names according to the number of their sides or angles, thus:

37. A pentagon is a polygon of five sides; a hexagon, of six sides; a heptagon, seven; an octagon, eight; a nonagon, nine; a decagon, ten; an undecagon, eleven; and a dodecagon, twelve sides.

38. A regular polygon has all its sides and all its angles equal. If they are not both equal, the polygon is irregular.

39. An equilateral triangle is, also, a regular figure of three sides, and the square is of four, the former being also called a trigon, and the latter a tetragon.

40. Any figure is equilateral when all its sides are equal. When both these are equal, it is a regular figure.

A circle is a plain figure bounded by a curved line, called the circumference, which is everywhere equidistant from the centre, Fig. 3.

The radius of a circle, Fig. 4, is a line drawn from the centre to the circumference.

The diameter of a circle, Fig. 5, is a line drawn through the centre, and terminating in the circumference on both sides.

A chord, Fig. 6, is a right line joining the extremities of an arc.

A segment is any part of a circle bounded by an arc and its chord, Fig. 6.

A semicircle is half the circle, or a segment cut off by a diameter, Fig. 5; the half circumference is sometimes called the semicircle.

A sector is any part of a circle which is bounded by an arc and two radii drawn to its extremities, Fig. 7.

A quadrant or quarter of a circle, Fig. 8, is a sector, having a quarter of the circumference for its arc, and its two sides are perpendicular to each other. A quarter of the circumference is sometimes called a quadrant.

The height, or altitude of a figure is a perpendicular let fall from an angle, Fig. 11, or its vertex to its base, or opposite side.

In a right angled triangle, Fig. 10, the side opposite the right angle is called the hypothenuse, and the two other sides are sometimes called legs, and sometimes the base and perpendicular.

When an angle is denoted by three letters, of which one stands at the angular point, and the other two at the two sides, that which stands on the angular point is read in the middle; thus, the angle contained by the line AC and EC, is called the angle ACE, or ECA. Fig. 9.

The circumference of every circle is supposed to be divided into 360 equal parts,

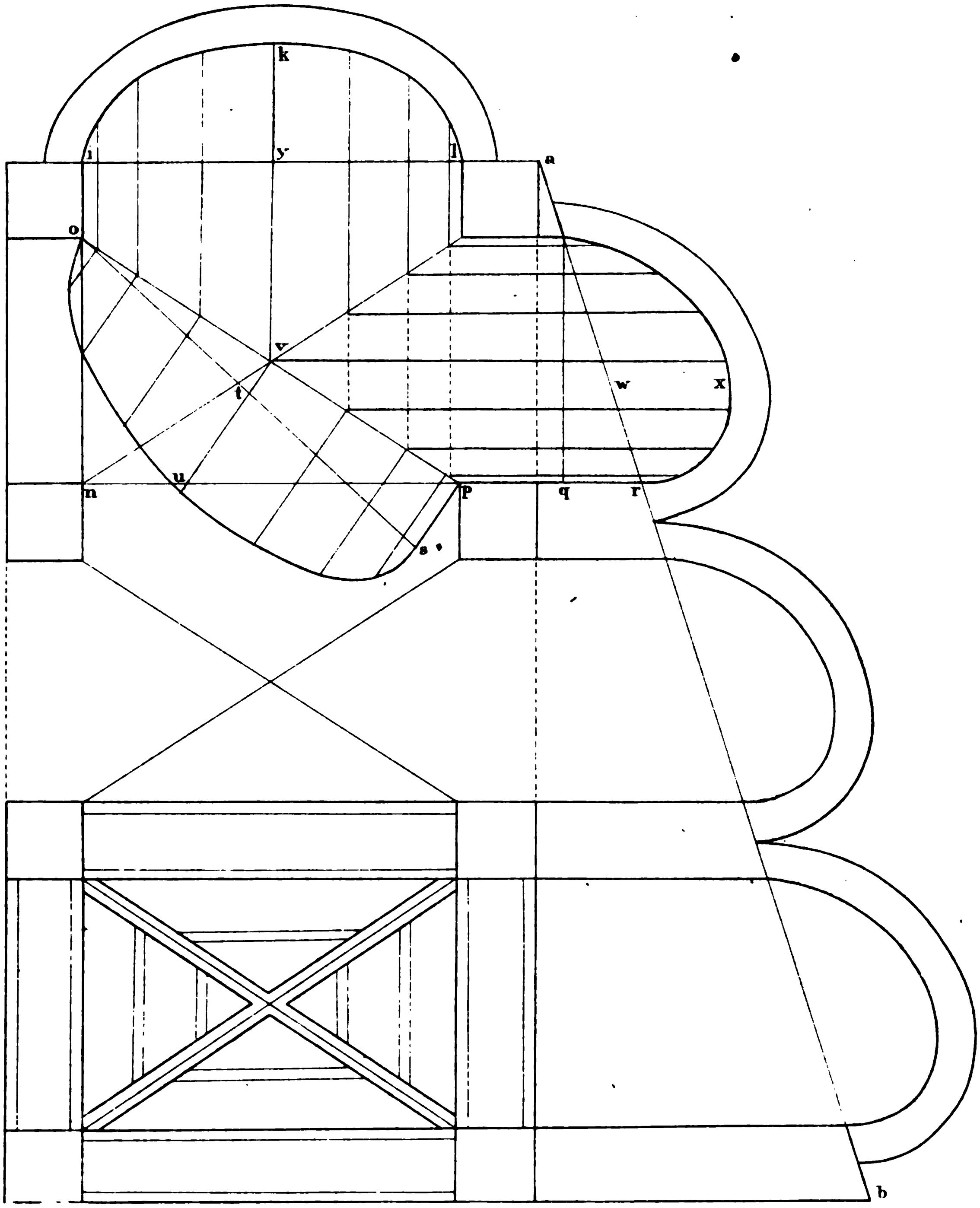

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PLATE IX.

Is a plastered groin arch designed for an oblong ceiling, and its angles represented by the letters A B C D. The angle, or groin ribs, are all connected at the centres of the ceiling and, consequently, may be termed an entire groin ceiling.

This design may be constructed of 1¼ inch boards.

To draw the ribs to an oblong plan, determine first, the height of the ceiling. In this design the groin rib at Fig. 2, is a semicircle; consequently, it raises one half of the smallest span, and the transverse and angle ribs will be a semi-ellipsis which may be drawn by two methods. One method is by ordinates, and transferring distances, which is both tedious and incorrect in practice. The other method is by a trammel and three strips of wood, as represented at Fig. 4, Plate IV.

Divide the given rib B 5 D, into any number of equal parts, say ten. From each point of division draw the dotted lines parallel to A B, and C D, to intersect the line C B in the points 1, 2, 3, 4, &c. Raise the perpendicular from the lines C B, and C D, and transfer the distances 1.1, 2.2, 3.3, &c., Fig. 2 to 1.1, 2.2, 3.3, &c., Figs. 1 and 3, and by tracing through those points, the other side and angle ribs will be found.

To draw Figure 4 with a Trammel.

First draw the line A B equal to the base of the transverse rib, Fig. 3: the rib to be drawn. On the upper side of the line, screw or nail a strip having one straight edge. At right angles screw two other pieces, leaving room for a small pin to pass freely between the pieces. Take any strip, as shown at C, and from the point 3 with 5.5, Fig. 3, as a distance, place a pin as shown at 4; and from the point 3, with C 5 as a distance, place the pin at 5. To describe the elliptic curve from 6 at the centre in the horizon, around to 1, at the base of the rib, suppose the trammel C, to be placed over and parallel to the line 6, 7; then by moving the trammel (taking care to keep the pin at 4, so that it shall slide against the strip,) around to 1, you will have described one half of the curve; and the other half may be obtained in the same manner.

NICHE.—In Architecture, a cavity, or hollow place in the thickness of a wall, in which to place a figure or statue.

Figure 2.

To describe a portion of a circle, by means of an angle, when the distance of the centre is very great.

Let A B be the length of the chord line, and D C the height of the required segment. Take two rods, C B and C A, each equal in length to the chord line, A B; cross the rods at the angle, C, and spread them out until the sides shall touch the points A and B; secure them at that angle by the cross-piece, E F; at the points A B fix pins, and at the angle C, place a point, or pencil; then move the pencil and rods around, taking care to keep the outsides of the rods pressed against the pin, and the segment A C B, will be described.

Figure 3.

To form an ellipsis by means of a string.

Let A B be the longest diameter, and D C one half the shortest diameter of the required ellipse. Make C F equal to D B, and E D equal to F D. At the points E and F, place pins; around the pins place a cord, so fastened at the ends that it shall reach around the points C, E, F; place your pencil inside the cord and describe the ellipsis. Care must be taken to keep the cord to an even tension.

Figure 4.

To make an ellipsis by means of a trammel.

Let A C, B D be the given diameters of the required ellipsis, and F G H the trammel, with pins at G H, and pencil at F. Make F G equal to half the shortest diameter, and F H equal to half the longest diameter. Provide grooves, into which the pins, G H, may slide, then move the trammel round, and an ellipsis will be described.

Figure 5.

To describe the false ellipsis, or an elliptical figure, by means of circular arcs.

Let A B be the length, and C D the breadth; join B D, and make B E equal to one third of B D; make E G perpendicular to B D; produce the line D C to G, then the points, F and G, with G D and F B for radii, will describe the required ellipsis.

Figure 6.

To describe an ellipsis by finding points in the curve.

Divide A E and A F, each into the same number of equal parts, as five, for example. Through the points of division, 1, 2, 3, 4 in A E, draw the lines B 1, B 2, etc.; and through the points of division 1, 2, 3, 4 in A F, draw the lines 1 D, 2 D, etc. Through the points of intersection draw the curve, and you will obtain one quarter of the required ellipsis.

PLATE V.

Figure 1.

Having the side of an octagon given, to describe the figure, and also to find a diagonal line with a carpenter's square.

Let A B be any line. Upon it place a common steel square, as represented in the figure, making 1, 2, equal to 2, 3. Draw the line 5, 6, parallel to A B, and equal to the given side; join 4, 5, and produce it to 7. Draw 6, 7, perpendicular to 5, 6; then 4, 5, 7, will be the diagonal; 5, 7, will determine the size of the octagon, and 6, 5, 7, will be the angle to cut the mitre.

Figure 2.

To describe a regular octagon from a given square.

Draw the diagonals; then with half the diagonals as a radius, describe from each of the four angular points of the square, the arcs cutting the sides. Join the opposite points of intersection, and the required octagon is obtained.

Figure 3.

A side of a polygon of any number of sides being given, to describe the polygon.

Let O A B be a line, and A B the given side of a polygon of eight sides. From A as a centre, with A B as a radius, describe a semicircle, and divide it into a number of parts corresponding with the number of sides in the required polygon. Through the points of division 2, 3, 4, etc., draw the lines A D, A E, etc. Perpendicular to O B draw B F, then A F will determine the size of the figure. Join B D E, etc., and the octagonal figure will be complete.

Figure 5.

To inscribe a polygon of six sides.

Describe a circle as represented, and, taking any point therein as a centre, with the same radius, describe arcs within the circle. Join the points of intersection by the chords, 1, 2, 3, 4, 5, 6, 1, and the required polygon will be obtained.

Figure 6.

To inscribe an equilateral triangle in a given circle.

Through the centre, A, draw any diameter, B C. From the point, C, as a centre, with the radius, C A, describe the arc, D A E. Join B D, D E and E B; then B D E is the equilateral triangle required.

PLATE VI.

Figure 1.

To find the section of a cylinder when it is cut by a plane perpendicular to the plane that would pass through the centre vertically.

Let the circle at A, represent one end of a cylinder, and the four sided figure 7, 8, 9, 10, its length; then 7, 10, will be the plane, cutting the cylinder perpendicular to the vertical plane that shall pass through the line 8, 9. Divide the semicircle 8, 7, 9, into any equal number of parts; draw the lines 1, 2, 3, etc., at A, to the line 7, 10, in B. Draw lines perpendicular to 7, 10, from the points 1, 2, 3, etc., on the line 7, 10, and equal in length to 1.1, 2.2, 3.3, at A. Describe the curve 7, 11, 10, passing through the points 1, 2, 3, etc., and figure B, is the section required.

Figure 2.

To cut a segment of a cylinder through three given points in the surface.

Let *a b*, Fig. 2, be the seats of the two given points, and *c* the given point in the plane. Join *a b*, and produce it to *f*, draw *a d* perpendicular to *a b*, and *b c* parallel to *a d*. Make *a d* equal to the height upon *a*, and *b e*, equal to the height upon *b*, join *d e*, and produce *d e* to *f*. Join *f c*, and produce *f c* to *g;* draw *g h* perpendicular to *f g;* make *a h* parallel to *c g;* produce *a h* to *i;* join *g i*, then *f g* will be the intersection, and *h i g* the inclination of the plane required.

It may be observed that instead of setting up the whole height of the points, we may set up one half, or one third, as in this case the heights over *a* and *b* are one half of the respective heights over each point. The section, A, is found by making the perpendiculars on *g i* equal to the corresponding ones, *g h*, and through the points thus found, trace the curve and you have the section required.

Figure 3.

To cut a semicylinder through a given plane, and oblique to its flat side, so that the oblique line shall form an obtuse angle equal to a given one.

Let 7 a 8 represent the semicircle; 9, 10 the plane, and 2, *c h* the obtuse angle. Produce *g c* to *f;* draw *e d* perpendicular to 9, 10, cutting the dotted line in *f*. Make *e d* equal to *c h*. Join *c d*, then *c d* will be the governing ordinate for the section. Divide 7, 8, in any number of parts. Take in the compass the distance, *c d*, and with one leg on *a*, describe an arc, cutting 7, 8, in *b;* then *a b* will be the governing ordinate for the plan. Draw the perpendicular lines from 1, 2, etc., on the line 7, 8, to the line 9, 10. Draw the ordinates, 1.1, 2.2, &c., parallel to the governing ordinates; Make 11 on the section equal to 11 on the plan; *c* 4, equal to 2.2, 2.2 equal to *b a*, and 3.3 on the plan. Describe the curve, 9 a 10, passing through the points 1, 4, 2, etc., and the required section will be obtained.

Figure 4

Exhibits the section of a hollow cylinder, and is obtained by the same method as in Fig. 2.

Centre, in building, is a combination of timber, so disposed as to form a frame, the convex side of which, when boarded over, shall correspond with the concavity of an arch of stone or brick during the time of erection.

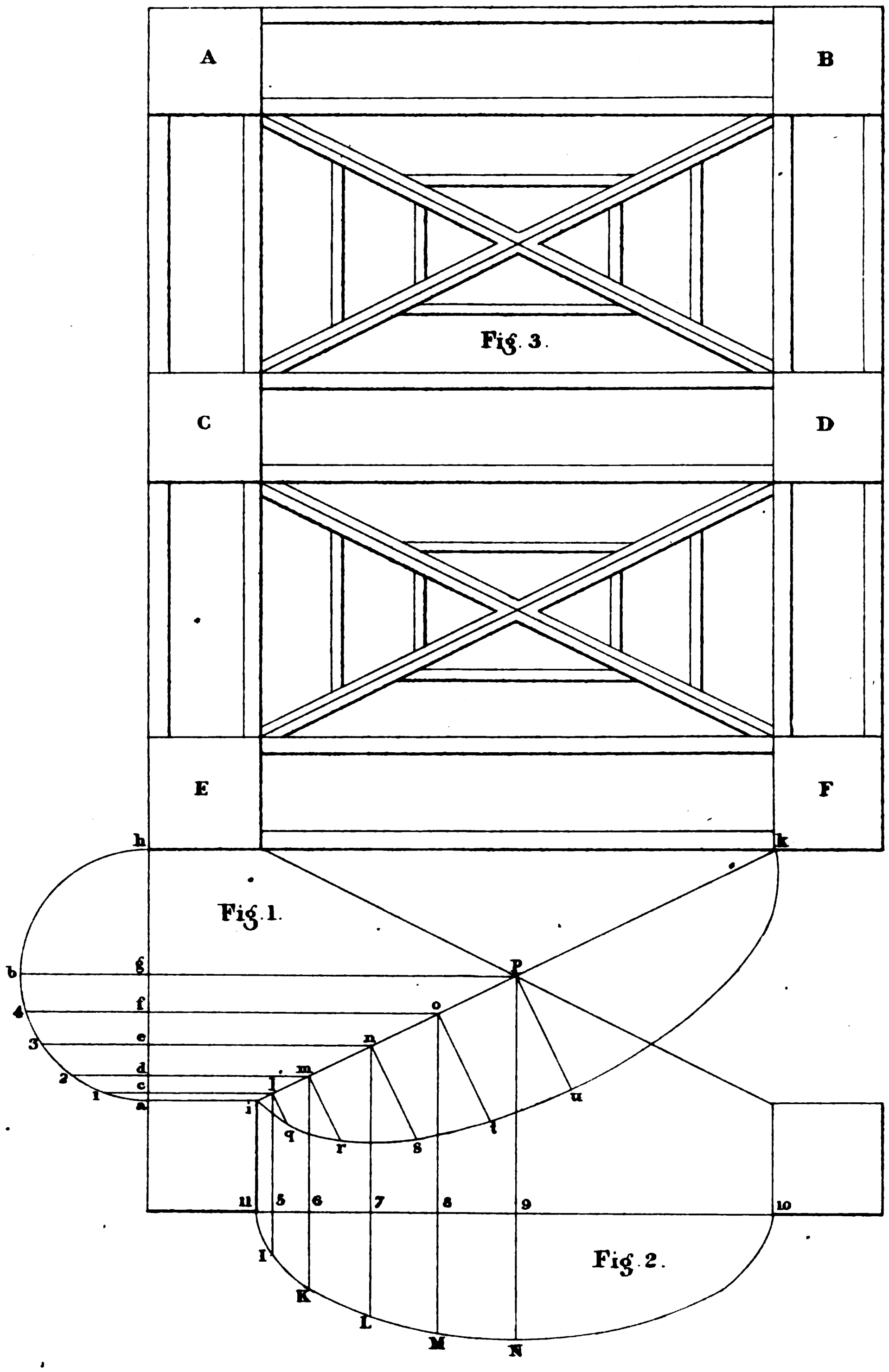

Fig. 3.

Fig. 1.

Fig. 2.

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Niche.—In Architecture, a cavity, or hollow place in the thickness of a wall, in which to place a figure or statue.

PLATE X.

Figure 1.

The plan of a niche in a circular wall being given, to find the front rib.

No. 1, is the given plan, which is a semicircle, whose diameter is *a b* and *a h b*, the front of the circular wall. Suppose the semicircle *a v*, *b* to be raised upon the points *a b*, so that the point *v*, may stand over the point *h*, in the front of the wall. The seat of the semicircle, standing in this position upon the plan, will be an ellipsis; divide, therefore, half the interior of the plan into any number of equal parts, say five. Draw the perpendiculars 1 *d*, 2 *e*, 3 *f*, &c., upon the centre, c. With the radius, *c h*, describe the quadrant, *h k*, which is divided into the same number of parts as the interior of the plan. Through the points 1, 2, 3, 4, draw the lines 1 *d*, 2 *c*, 3 *f*, 4 *g*, parallel to *a b*, intersecting the lines 1 *d*, 2 *e*, &c., in the points *d e f*, &c. Find the stretch-out of the inner arc at No. 2, by the method shown at Plate III., *Fig.* 6. Make *a b* at No. 3, equal to the line *a b*, No. 2. Divide the line *a b*, No. 1, into ten equal parts. Take the distance intercepted between the curve line, *a h b*, No. 1, and the points, *d e f g*, and set them on the lines, 1 *d*, 2 *e*, 3 *f*, &c., No. 3, making the production of each line to correspond with the different distances; and, through the extreme points, trace the curve for the mould to be applied to the underside of the front rib, so that the edge of the front rib will be perpendicular to the plan.

C D E, are back ribs shown separately.

Fig 1

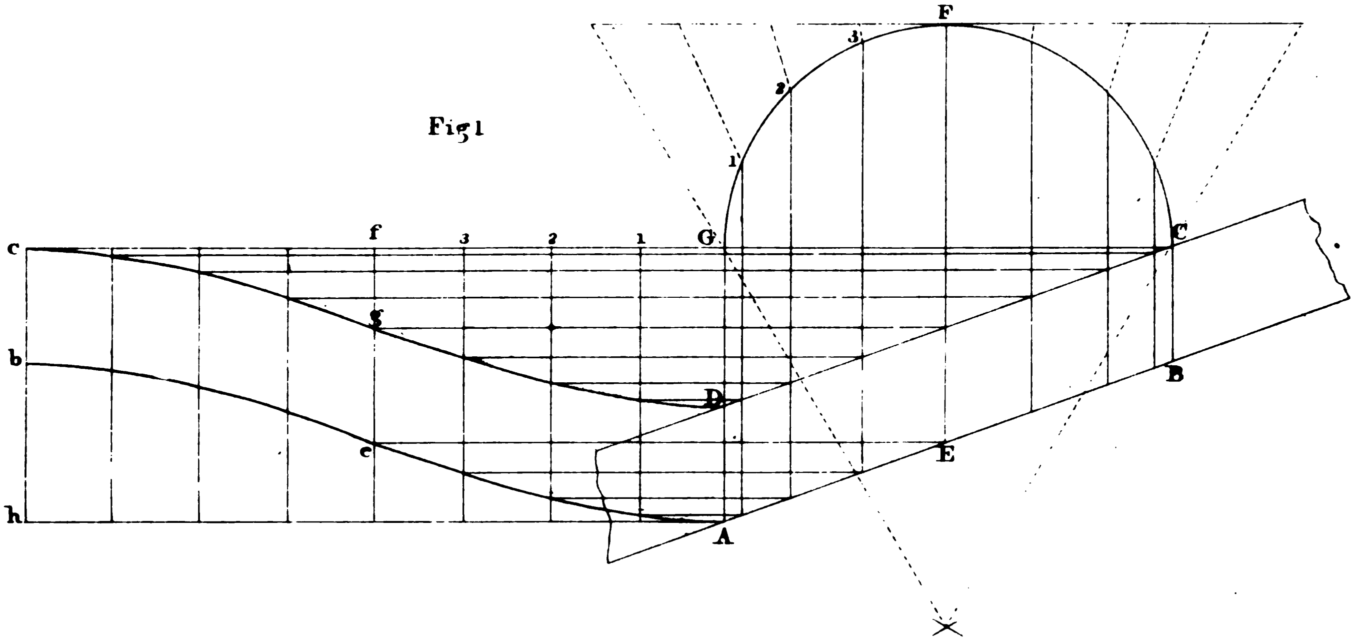


Fig 2

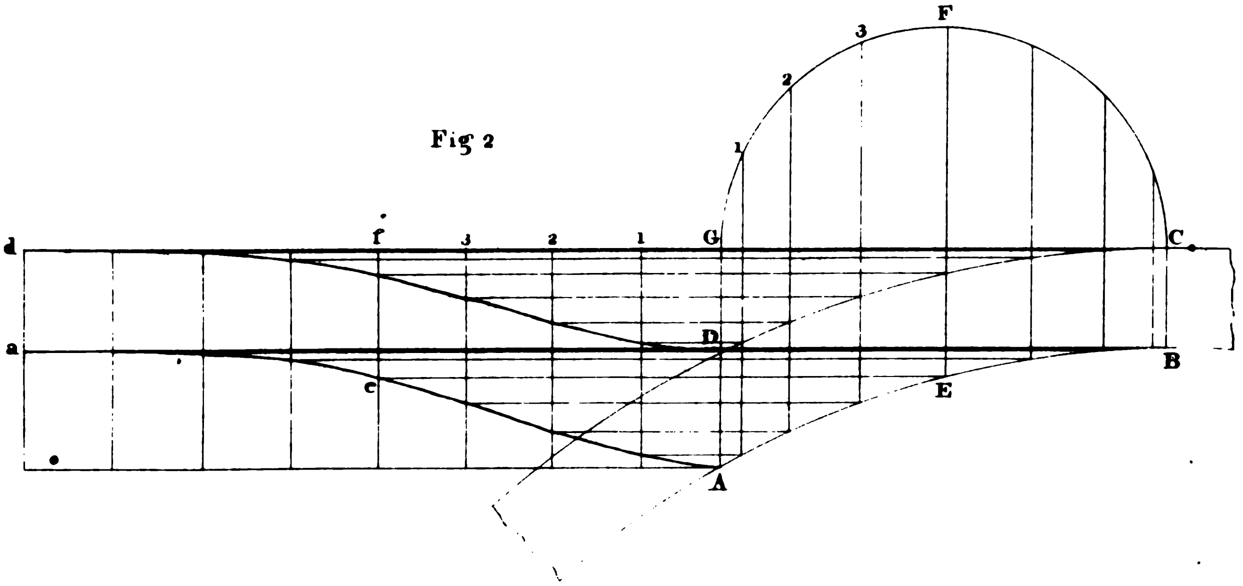
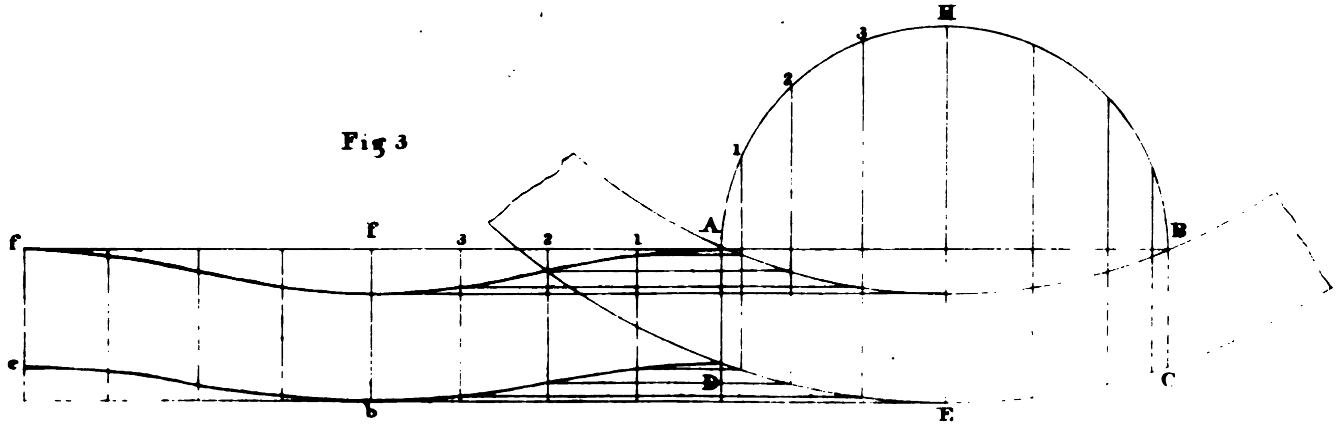
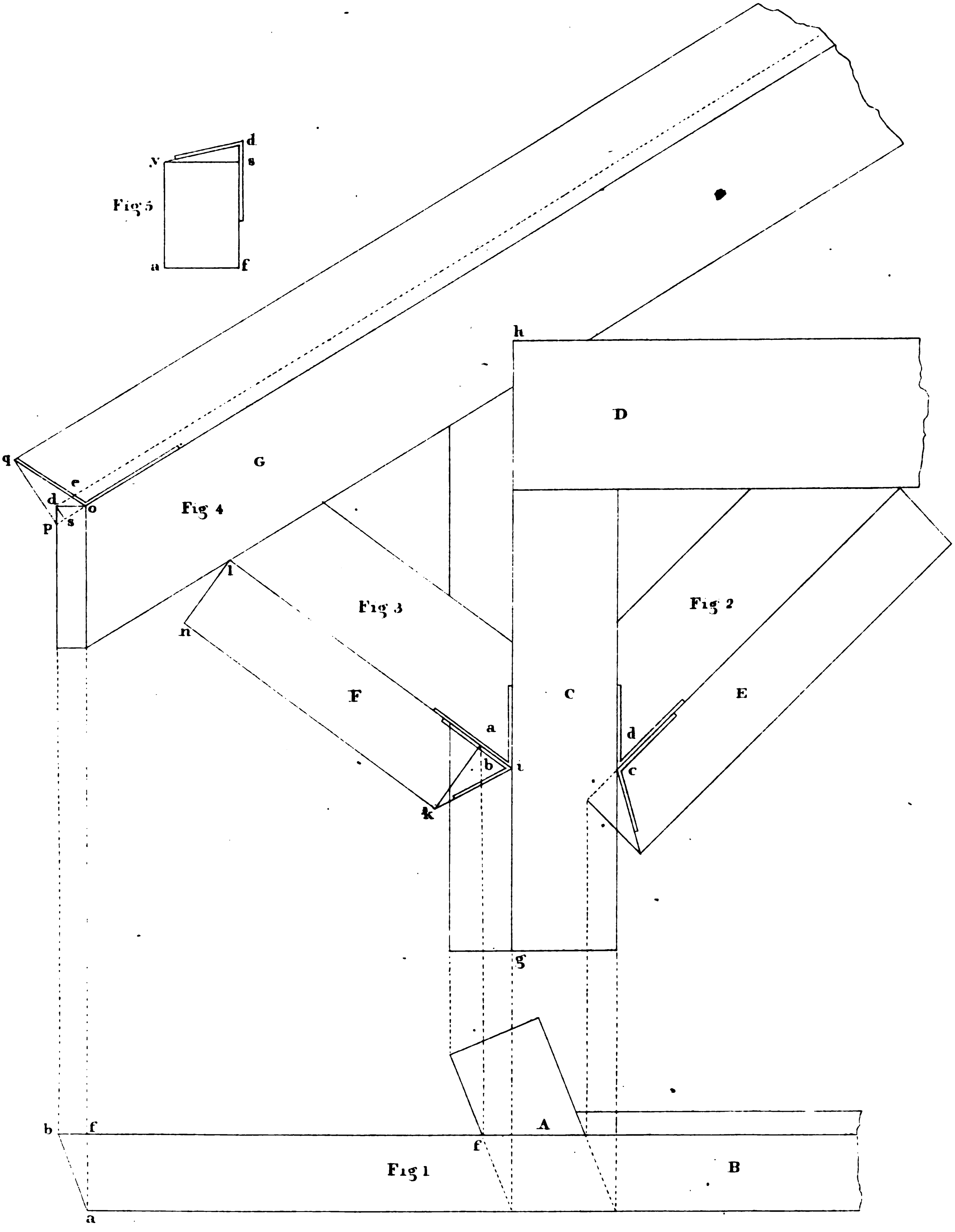


Fig 3



D. McLeish Lith. Syracuse N.Y.



Invented and Drawn by L. D. Gould.

D McLellan Lith 26 Spruce St N Y

Figure 2.

To develop the soffit of a circular arch which cuts obliquely through a circular wall.

Let A B C D be the plan of the wall. Draw C G *d* perpendicular to C B, and produce A D to meet C G *d*, in G. On C G, as a base, describe the arc, G F C; divide the arc into equal parts, and extend those parts on the line G *f d*, making G *f d* equal in length to the stretch-out of the curve, G F C. From each point of division draw a line parallel to G A, and from the points where the division lines cut the lines of the wall in the plan, draw lines parallel to *d* G C, and through the points of intersection, with the division lines in the development, trace the form of the soffit required.

Figure 3.

To find the soffit of a circular door-head, or window cut through a circular wall.

Draw the lines B A *f*, at right angles to the sides, or jamb lines, B C and A D. Make A *f* equal in length to the stretch-out of the curve, A H B, as at Figs. 1 and 2, and in every respect the process is precisely the same as in the examples shown at Figs. 1 and 2.

BRACKETING.

Bracketing is the method of forming the angle between the ceiling and the side walls of a room for the lath and plaster cornice. Its form may be elliptical, or of other compound curves; and when they are large, in order to save materials, the plaster is supported on laths which are fastened to wooden brackets, placed from twelve to sixteen inches from centres; and in order to support the laths at the mitres, brackets are fixed at the internal and external angles.

PLATE XIII.

Figure 1.

Shows the method of finding the bracket for an external acute angle.

Let A C be the projection of the cove; C *b* the line of the side wall; *b f* the line of the ceiling, and *b* C D the given external angle. Draw A E, and E G parallel to C *b* and O D. Make A *f* equal to A C. Divide the given bracket into any number of equal parts, and from the points of division on the curve line drop the lines 1.1.1, 2.2.2, &c., parallel to *f* A E, cutting the line E O in the figures 1, 2, &c. Draw E *h* perpendicular to E O, and equal to A *g*. Draw 1.1, 2.2, &c., parallel to E *h*, and make the lengths of the perpendiculars respectively equal to those opposite. Between the base A C, and the curve *g* 1, 2, &c., and through the points thus found, draw the curve of the angle rib required. The dotted curve shows the bevel, or splay of the bracket, to form the plane that shall coincide with the plane of the given brackets.

Figure 2

Exhibits the method of finding the bracket for an internal right angle, and is precisely the same as Fig. 1, *with the exception of bevelling the angle bracket, which is not necessary for an internal angle.*

Plate 14

Fig 1

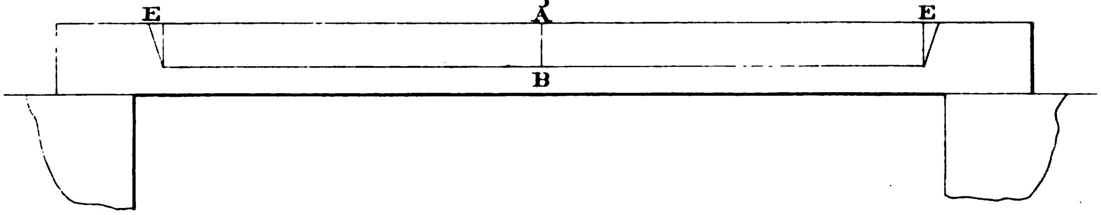


Fig 2

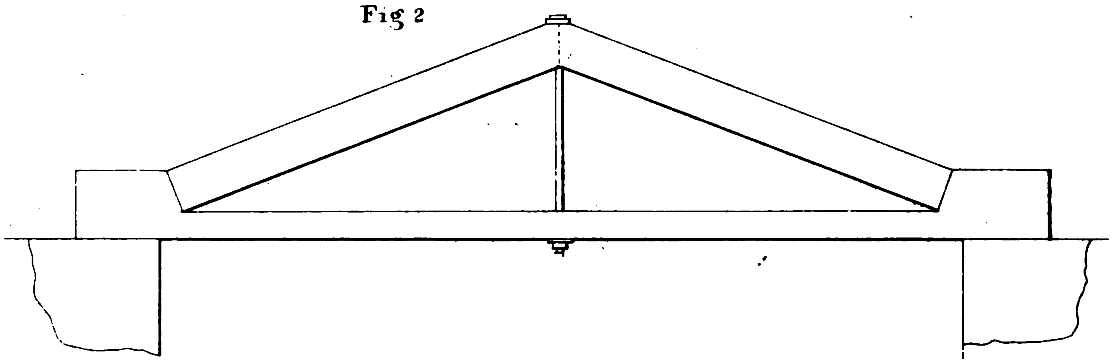


Fig 3

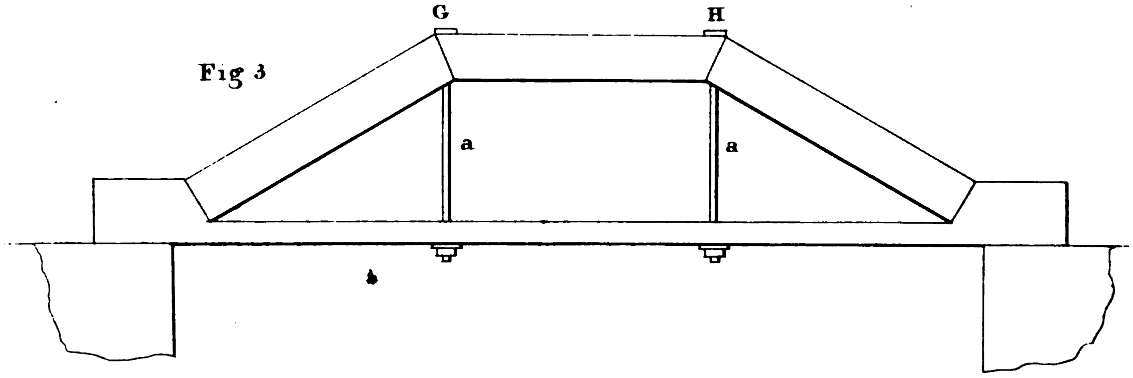


Fig 4

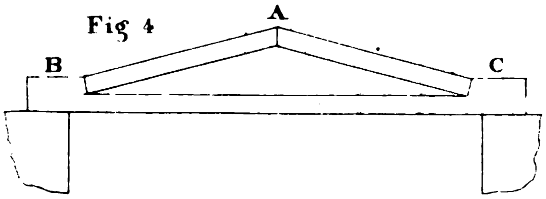
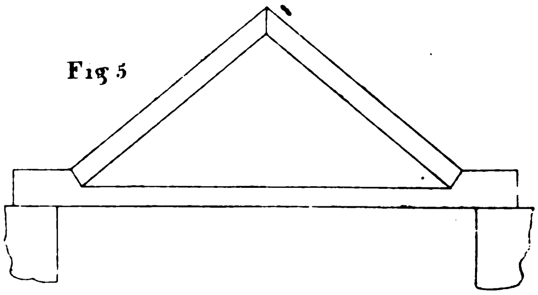


Fig 5



D. M^c Lellan Lith. 26 Spruce St. N.Y.

PLATE XIV.

Let Fig. 1 be a piece of timber cut in the various directions indicated by the lines passing through it; and let the triangular pieces shown at E and F be removed. Then raise the pieces, A E and A F, till they make close joints at E and F; and increase their lengths till they form a frame, or truss, as represented at Fig. 2. A small rod of iron, with suitable nuts, will be required to support the centre of the tie, as seen in the drawing. If the depth of the frame at the middle be double the depth of the beam, the strength of the frame will be a little more than three times the strength of the beam, and its firmness will be a little more than eight times as great as that of the beam. If the depth of the frame be three times the depth of the beam, as represented at Fig. 2, it will be about six times as strong as the beam, and about eighteen times as firm, that is, it will bend only an eighteenth part of the distance which the beam would bend under the same weight.

To render the strength more equal, and to obtain two points of support, there may be a level piece of timber placed between the inclining ones, as shown at Fig. 3, but if a greater weight be placed at G, than at H, there will be a tendency to spring outwards at H, and inwards at G, which may be effectually prevented by the suspension rods, *a a*, as shown in the same figure.

It now remains to show why the strength of a piece of timber is increased by forming it into a truss; and to have a clear conception of this subject, is of the utmost importance in the science of carpentry.

Let A B C, Fig. 4, be a truss to support a weight applied at A. It is evident that the force of the weight will tend to spread the abutments, B and C; and the nearer we make the angle, A B C, to a straight line, the greater will be the pressure, or tendency to spread, or increase the distance between the abutments, B and C, by the same load at A. On the contrary, if the height be increased, as at Fig. 5, the tendency to spread the abutment will be less.

The advantage of framing timbers together for the purpose of giving strength and firmness, having been shown, let us proceed to explain how the strain on any part may be measured.

PLATE XV.

Figure 1.

To find the pressure on oblique supports, or parts of trusses, frames, &c.

Let A B be a heavy beam, supported by two posts, A C and B D, placed at equal distances from E, the centre of the beam. The pressure on each post will, obviously, be equal to half the weight of the beam. But if the posts be placed obliquely, as in Fig. 2, the pressure on each post will be increased in the same proportion as its length is increased, the height A C, being the same as before; that is, when A F is double A C, the pressure on the post in the direction of its length is double the half weight of the beam. Hence it is very easy to find the pressure in the direction of an inclined strut, for it is as many times half the weight supported as A C is contained in A F. Therefore, if the depth A C, of a truss to support a weight of two tons be only one foot, and A F be ten feet, the pressure in the direction of A F will be ten tons.

It will be observed that when the beam is supported by oblique posts, as in Fig. 2, these posts will slide out at the bottom, and together at the top, if not prevented by proper abutments. The force with which the foot F, tends to slide out, is to half the weight of the beam A B, as F C is to A C. Therefore, when F C is equal to A C, the tendency to slide out is equal to half the weight supported; and if F C be ten times A C, the tendency to spread out would be ten times the weight supported. Hence, it is evident that a flat truss requires a tie of immense strength to prevent it from spreading. If a flat truss produces any degree of stretching in the tie, the truss must obviously settle; and by settling it becomes more flat, and consequently exerts a greater strain. In a flat truss, therefore, too much caution cannot be used in fitting the joints and choosing good materials.

When the spreading of a frame is to be counteracted, it is most effectually done by a straight tie-beam connecting the points together; but sometimes a carpenter is so limited as to the space in which the truss is to be formed, that he cannot obtain a straight tie, and then it is desirable to know the strain on such a tie as can be procured.

Figure 3.

Let ACD be a truss, to which a straight tie AED, cannot be applied without interfering with the architect's design. In this case, let AB and BD be the ties. Draw the lines in the centre of the pieces forming a triangle, ACB. Then as CB is to AB, so is half the load at C, to the strain it produces on the tie AB; and the strength of the frame ABCD, is to the strength of a frame of the same quantity of material having a straight tie, ACD, as CB is to CE. In the example we have drawn, the frame would have only one third the strength of one with a straight tie.

It is a serious objection to this kind of framing, that any settling, however small it may be, tends to spread out its supports, as represented at A and D, while a straight tie has scarcely any other sensible effect than that it draws the supports together, which is less objectionable than spreading them apart.

Figure 4

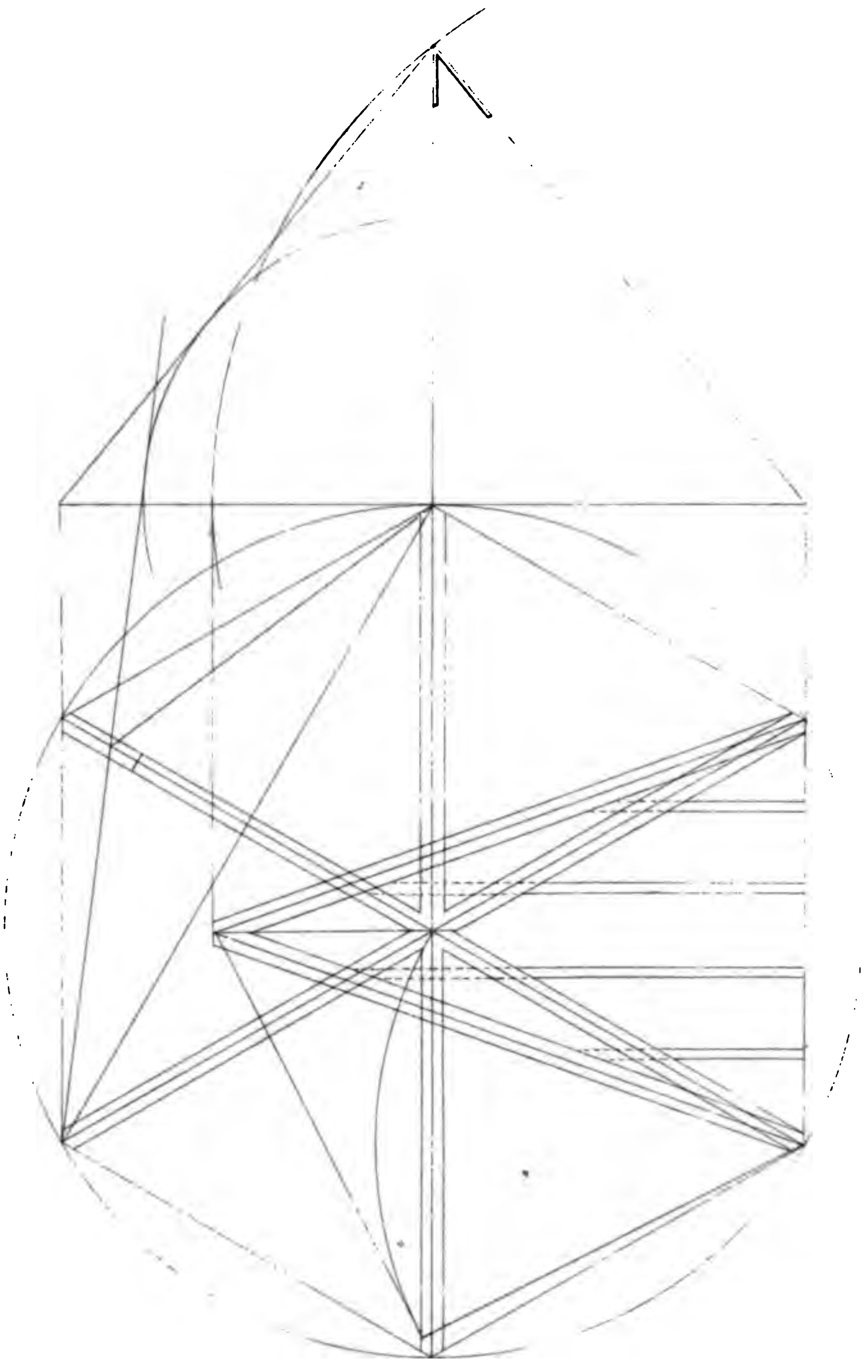
Exhibits a very cheap and expeditious plan for framing a roof to span from 40 to 70 feet. It requires no explanation further than to say that the tie need not be more than 5 × 10 inches; the rafters and braces 5 × 8; the battens of 1 inch boards, spiked to the timbers with large nails. It is believed to be the best roof that can be constructed, as it has all the advantages of a solid mass, without the great weight and the disadvantages of the shrinkage of material, which is almost entirely obviated by the crossing of the fibres of the wood.

OBSERVATIONS ON THE FORMS OF ROOFS.

All gable-ended roofs have a tendency to spread out the walls, particularly when the walls are thin, or when they are a great distance apart.

In the construction of roofs of every description, the ties extending from one plate to another, or to the wall plates, should be well connected at the angular points, for when the building is carried up on a polygonal plan, every two adjacent, or opposite hips, will act in the same manner as the two principals in a framed roof.

Plate 26



Invented and Drawn by L.D. Gould.

[illegible]

with it. Take any distance in the compass, and from the point, *s*, describe the circle, *m r y q*. Draw the lines 1, 2, 3, parallel to the wall lines, D C, to meet the lines E C, of the plan of the hip-rafter. Draw *m i*, *r t*, *y x*, and *q z* parallel to D C. Perpendicular to *r t*, draw *t i*; and perpendicular to *xy*, draw *x z*. Join *u i* and *u z*, then the angles *w u z* and *w u i*, will be the angles at which to cut the two sides of the purlin, in order to fit against the hip-rafter. The bevels, R K, are properly applied, K being the face, and R the down bevel.

3d. *To find the lengths of the jack-rafters, together with the down and face bevels for the hip-rafters.*

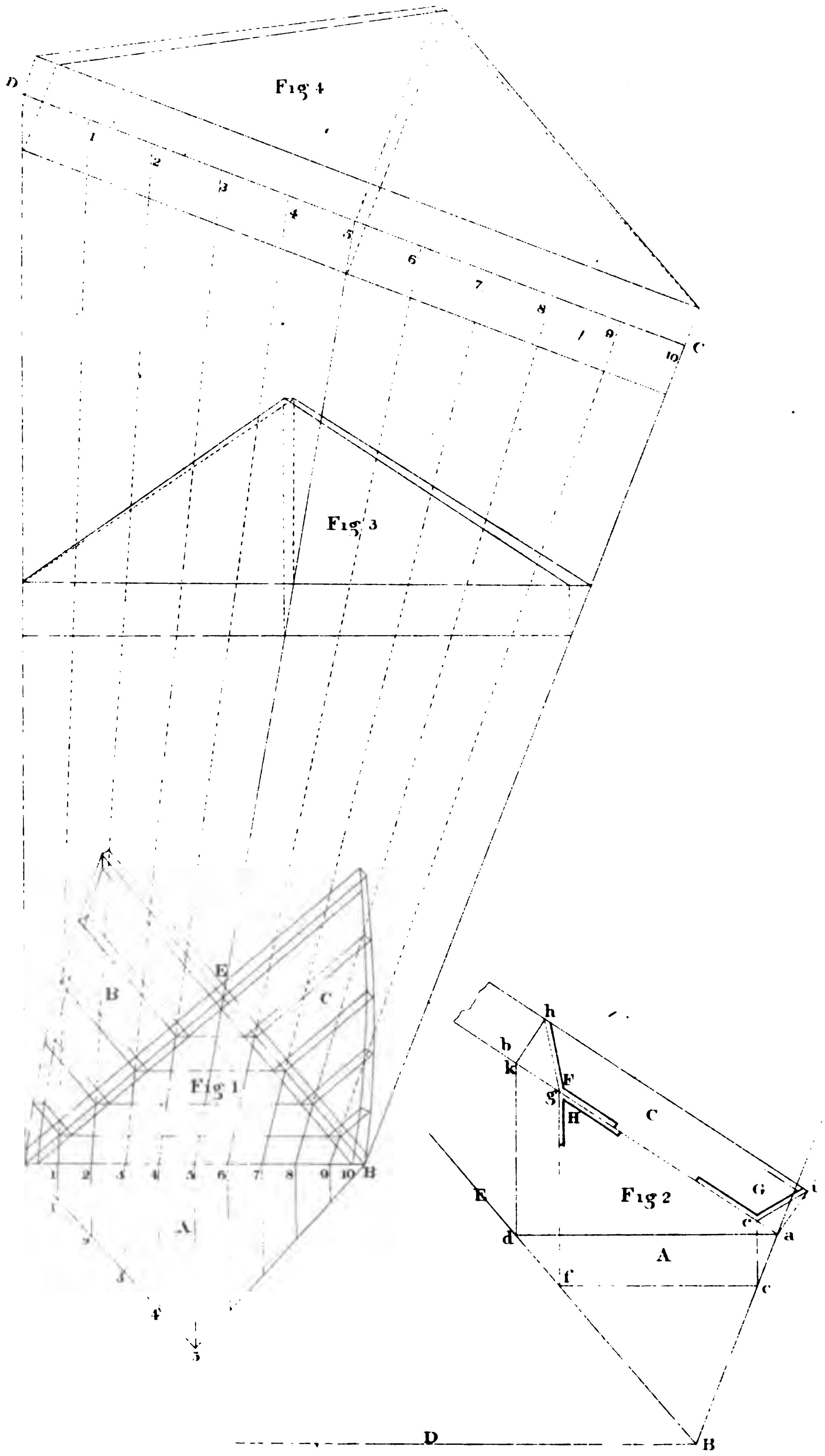
Divide the line G H, into as many parts as you have jack-rafters; then 1.1, 2.2, &c., will be the lengths of the jack-rafters required, and the bevel, V, will be the down bevel, and the bevel, Q, will be the face-bevel. G H B represents the hip, and jack-rafters put together, and ready to raise up to their place. Make B O equal to B E, and O H equal to *g f*, then the bevel V, will be the down, and W, the face-bevel, for the hip-rafters.

PLATE XVII

Exhibits the application of the foregoing principle to obtuse and acute angles; and the same process is observed as in Plate XVI. The cuts of the purlin are not shown because the method of finding them is the same as in the preceding plate; and every thing is so plain that a bare inspection of the Figure is sufficient for its comprehension.

PLATE XVIII

Is a plan and elevation of a roof framed to an obtuse and acute angle. The plan being rhomboid, the projection of the rafters is supported by braces. The lines for finding the shoulders of the braces to fit the obtuse and acute angles; the section, or backing of the rafters, and the bevels, to find the vertical and face-joints, so that their ends shall be in the same plane, will be found on a large scale in the following plate.



Invented and Drawn by L.D Gould

D. Mc Lellan Lith 26 Spruce St. N.Y.

The plan of the roof, Plate XX, is indicated by the letters A B C D; *a b*, Fig. 1, and *c d*, Fig. 2, are sections of the purlins. Fig. 1 is an elevation of the roof, standing over the line A B, and Fig. 2 is an elevation over D C, and shows the method of framing the principal rafters.

PLATE XXI

Shows the method of finding the hip-rafters for one end of the plan, the backing of the hips and common rafters, and the cuts for the jack-rafters.

To find the hip-rafters, divide the lines A B and D C, of the plan, into any number of equal parts, as in this case, ten. Join 1.1, 2.2, 3.3, &c., by the dotted lines 1.1, 2.2, 3.3, &c. Join the intersection of the dotted lines with the plan of the hips A E, and E B, by the lines running parallel to A B. From the angles on the plan of the hips, and at right angles with A E and E B, raise perpendicular lines, and make them equal to the corresponding lines at A, and through the points thus found describe the curves, which will be the form and backing of the required hips.

To find the cuts of the jack-rafters E E, Plate XX, make the angle B, Fig. 2, equal to the angle B, of the plan, Plate XX. Lay the plan of the hip and jack-rafter down in the same position, in respect to the angle B, as shown at A. From the line *d a*, of the plan, make the line *a b*, the elevation of the jack-rafter. Draw lines parallel to *a d* and *a b*, and make the spaces contained between the parallels each equal to the breadth of the rafter. Draw *d k f g* and *c e*, perpendicular to *a d*. Draw *a i* and *k h*, perpendicular to *a b;* join *e i* and *g h;* then the bevel G, will be the foot of the rafter; F will be the face, and H the down bevel of the butt-joint against the hip-rafter. The bevels thus found must be applied before the rafter is beveled to the plane of the roof.

Figures 3 and 4, show the amount of wood to be taken off, to form the backs of the common rafters, and this is indicated by the diverging lines shown on the elevation of the rafters. The joints, at the top, or peak, of the roof, are found in the same manner as at G, Fig. 2.

Fig 2

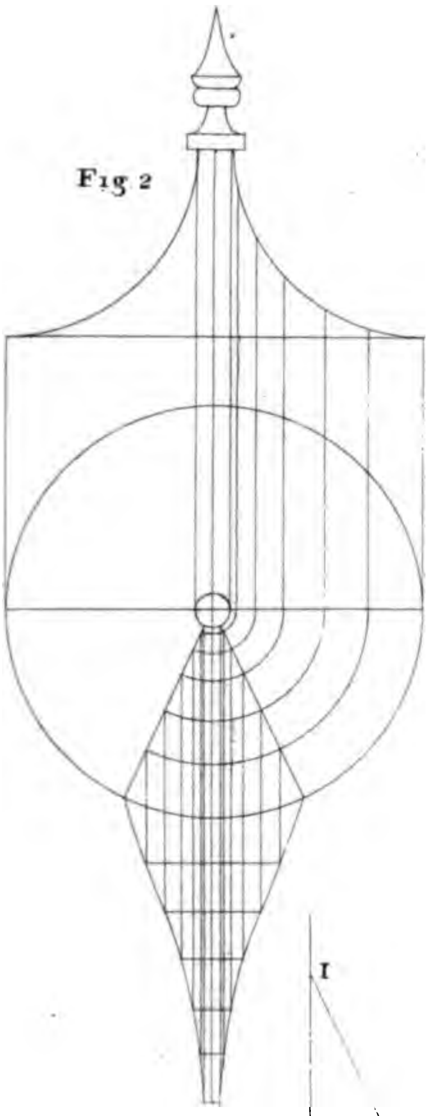


Fig 1

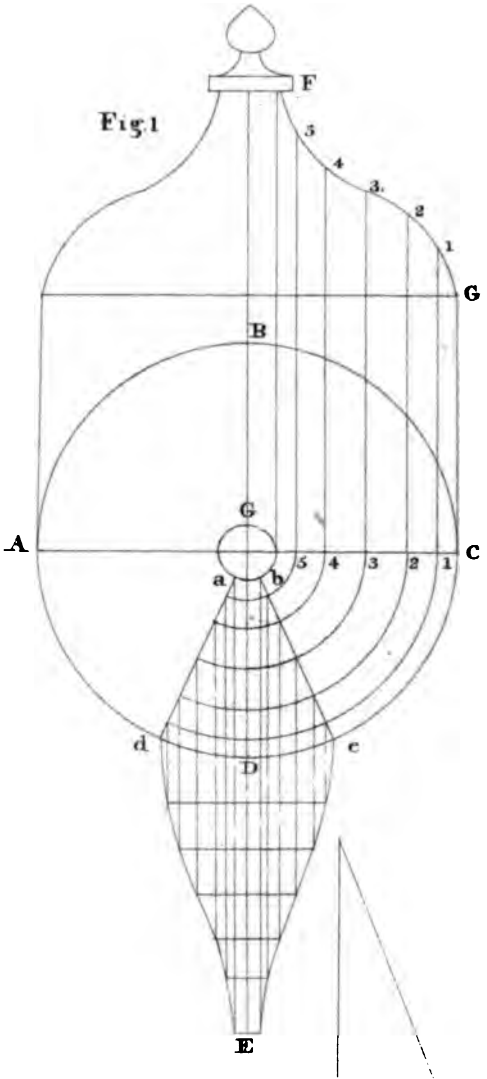


Fig 3

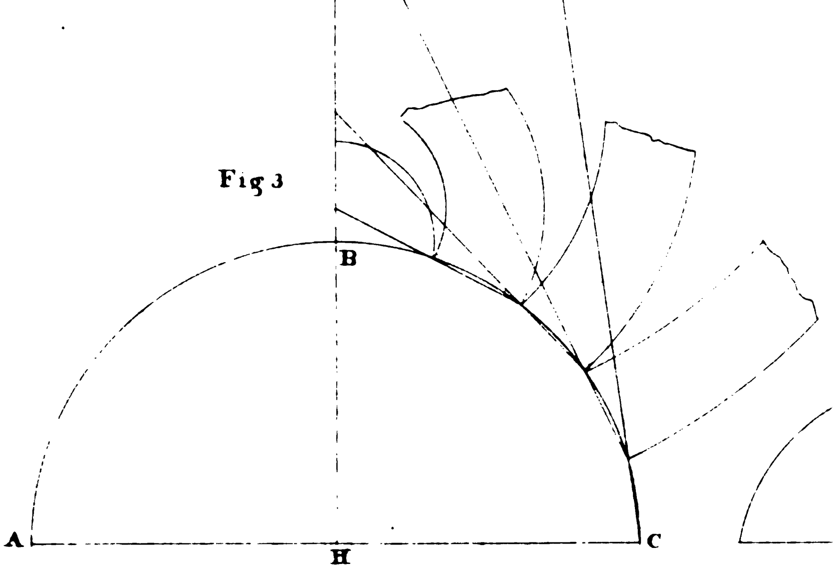


Fig 4

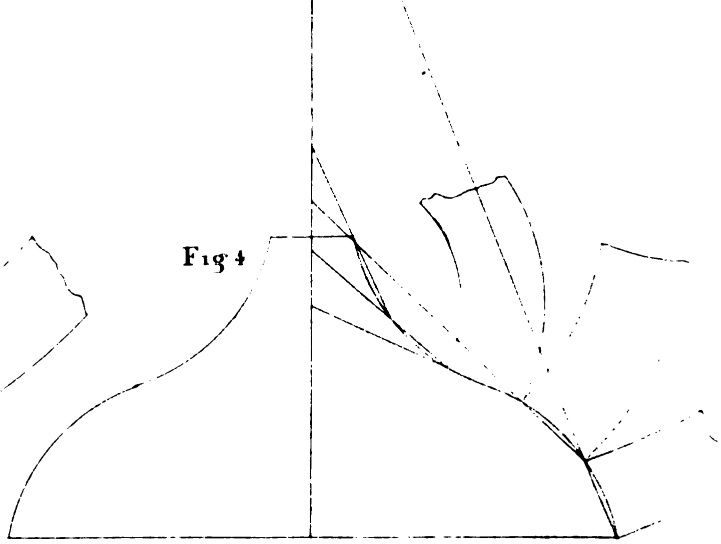


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Fig 1.

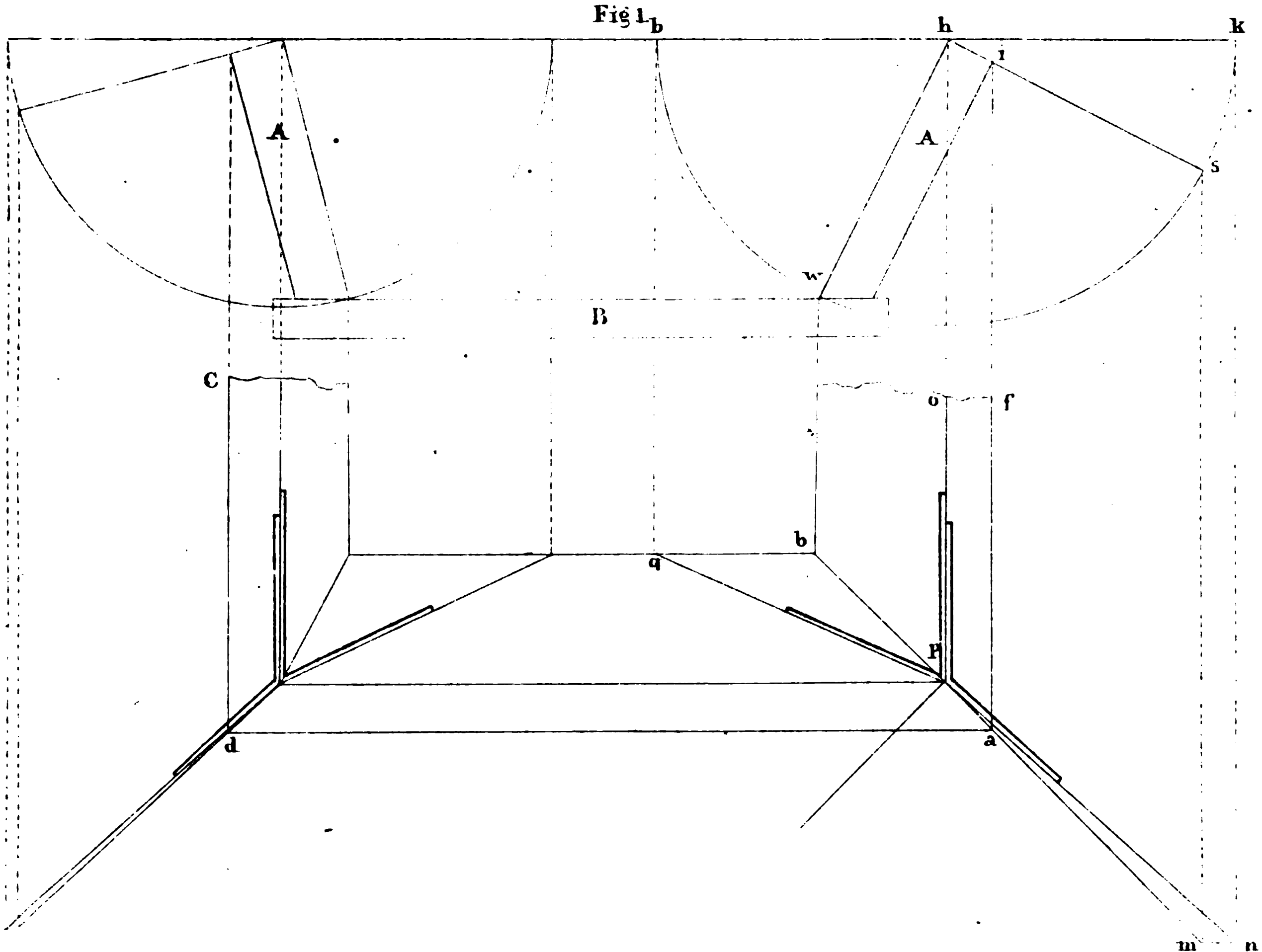
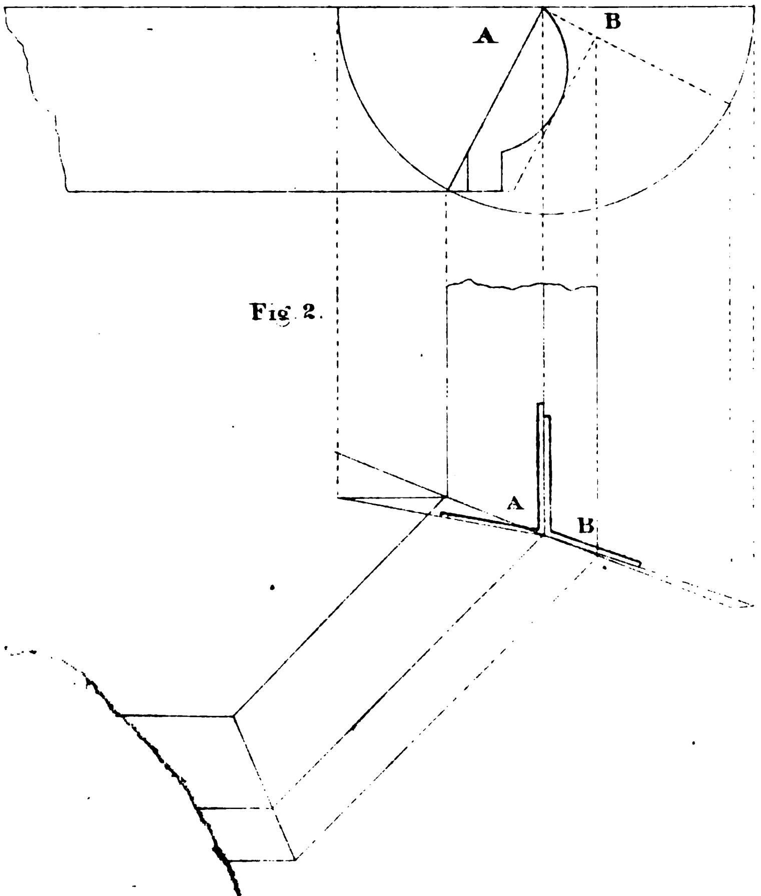


Fig. 2.



Invented and Drawn by L.D. Gould.

D. Mc Lellan Lith. [illegible]

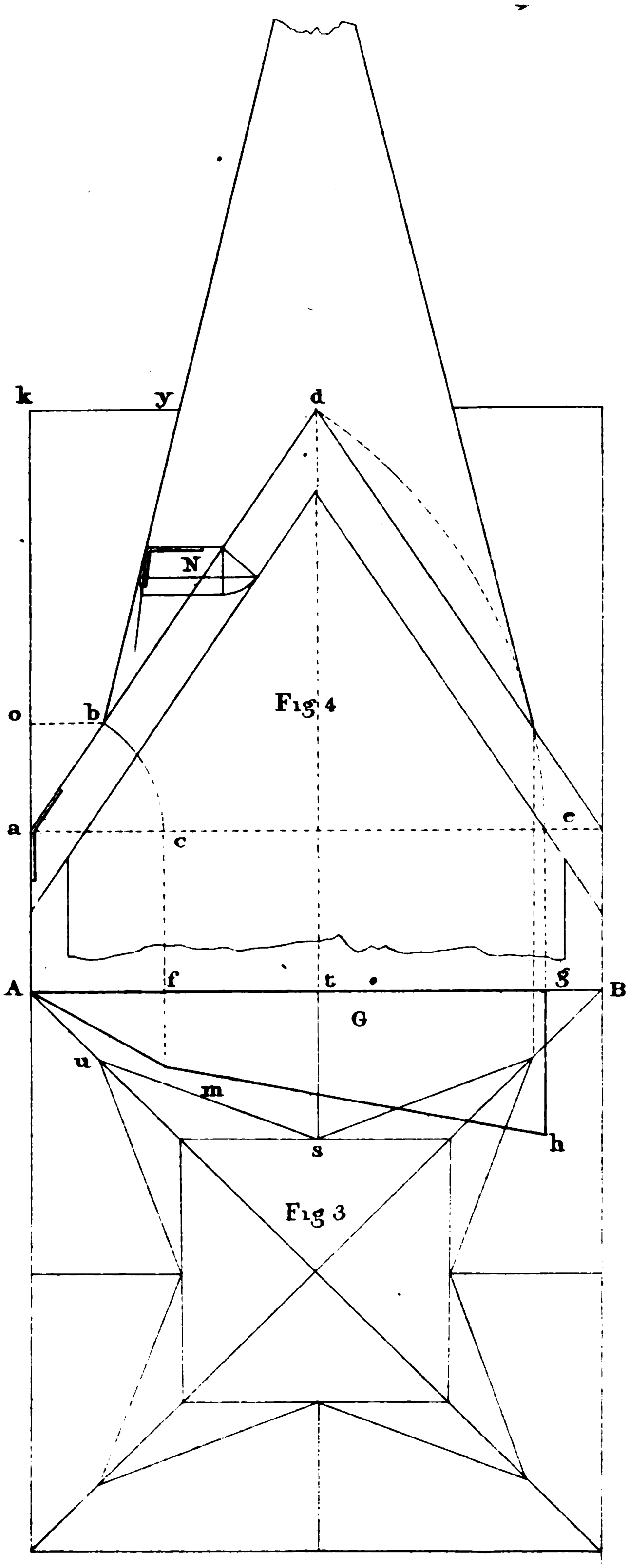
diameter BD, at right angles to AC. Produce BD to E, and equal in length to the curve line GF. Divide the curve line GF, and the straight line DE, into the same number of equal parts, and from the points 1, 2, 3, &c., on the curve line GF, drop the lines parallel to GC, to meet the line AC, in the points 1, 2, 3, &c. With one point of the compass on the centre of the plan at G, produce the parallel lines 1.1, 2.2, 3.3, &c., around and across the plan of the covering *abcd;* from the points of intersection, draw the parallel lines to meet those in the line ED, and through the points thus found trace the form of the board.

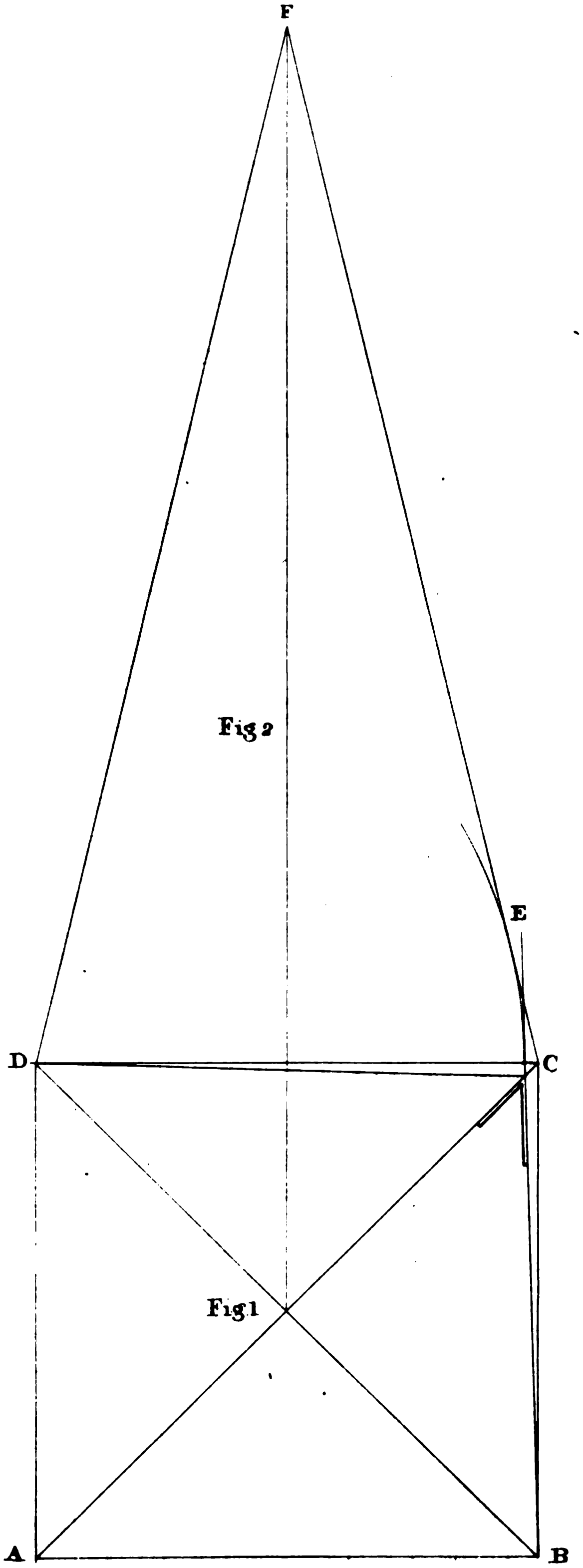
In Fig. 2, the form of the mould for the boards is found in a similar manner.

The method just described may be called the vertical method; that which is now about to be given is called the horizontal method.

Let ABC, Fig. 3, be a vertical section of a circular dome; and let it be required to cover this dome horizontally. Bisect the base AC, in the point H, and draw HI perpendicular to AC, cutting the semi-circumference in B. Divide the arc BC, into as many parts as it will require boards to cover the dome. Draw a line through the two points of division, producing it till it touches the perpendicular, HI; then the line intercepted between the extreme point and the perpendicular HI, will be the radius to describe the board to cover that part of the dome contained between the two points. The same process will be required for the other boards until the covering shall be completed.

Fig. 4, is covered in the same manner as Fig. 3.





Invented and Drawn by L. D. Gould.
D. McLellan Lith 26 Spruce St N Y

facilitated by applying the bevels to a box, the down bevel being laid on the side, and the bevel contained in the angle *m* A *f*, on the top of the mitre-box.

PLATE XXXII.

Figure 1.

Shows the method of finding the lines on the face and butt-joints of a piece set to a given plane with the base, to mitre over an acute, obtuse, or right angle.

Let A B C be the plan of an acute angle; B D the line of intersection; A *b c d* the piece; A *d* the plane, and A *f* the base of the plane. To find the face-joint, make A *f* equal to A *d*. Draw *d h* parallel to A B, cutting the line B D in *h*. Perpendicular to *d h* draw *h g*. Join *f g*, making the line *f g* parallel to *d h*. Join B *g*; then the angle A B *g*, contains the bevel for the side A *d*, of the piece to mitre over the acute angle, A B C.

To find the butt-joint, or the line over the edge *d c*, of the piece, make *c y* equal to *c d*; draw *y x* parallel to *d h*; produce *g h* to cut the line *y x* in *x*; join *x* D, then *y x* D is the angle, and contains the bevel for the side *d c*, of the piece to mitre over the angle A B C, of the plan.

Figure 2.

Exhibits the application of the same principle to an obtuse angle, and a bare inspection of it will show that the process is precisely the same as in Fig. 1.

PLATE XXXIII.

Figure 1.

Shows the method of mitering together a splayed box, also to find the cuts in a mitre box, by which to cut a moulding that a bevel cannot be applied to conveniently.

Let A A be the sides, B the bottom, and *c d a f*, the plan of the box. With one point of the compass on *h* and the other on *v*, describe the arc, *b v s k*, and from the points *b v s k*, perpendicular to *b k*, draw the lines *b q*, *v b*, *s m*, and *k n*.

Join $b\,a$, and produce $b\,a$ to m; then $b\,a$ is the mitre, or intersecting line on the plan. Draw $m\,n$ perpendicular to $s\,m$; join $p\,n$ and $p\,q$; then the angle $o\,p\,n$ contains the bevel at which to cut the upper edge of the box at $h\,i$; and the angle $o\,p\,q$ contains the bevel for the out and inside of the piece, A. The other side of the figure shows the application of the same principle where the sides are not of the same bevel, the operation being performed on both sides of the box to find the lines for the two different bevels.

Figure 2.

Shows the same principle applied to an octagon figure, the mitre box being used to cut the mitres, either for facilitating the work, or when the surface of the moulding is such that a bevel cannot be employed. In such a case, A will be the bevel to apply to the top of the box, and B the bevel applied to its side. In placing the piece in the box, lay the side A, downwards.

PLATE XXXIV.

Shows the method of finding the lines for mitering together a grain-mill hopper at its angles; also to find the angle pieces that are required to secure the pieces together at their intersection.

Let A B C D, Fig. 1, be the plan of the angular box, and Fig. 2 the elevation. In Fig. 2 place one point of the compass on a, and with the radius $a\,b$, describe the dotted curve $b\,d$; then $a\,d$ will be equal to $a\,b$. Draw the dotted line $d\,f$ perpendicular to $a\,d$, to the line $f\,g$ on the plan, Fig. 1, cutting the line $f\,g$ in f. Join Bf and Cf; then B Cf is the development of one side of the figure, and consequently contains all the angles for constructing a box of any size.

It is usual to place the sides of all mill-hopper boxes at an angle of 45°. It may, however, be desirable to make the elevation of the sides greater or less than 45°, and it is therefore proper to give the method of finding the butt-joint for any angle, from a horizontal to a perpendicular.

To find the mitre, or butt-joint across the edge of one of the sides of the box: Produce *d a* to *i.* Make *a i* equal to *a h.* Produce the diagonal line A C, until it cuts the line *h m* in *m.* Through the point *m,* perpendicular to *h n,* and parallel to D C, draw *n m h.* Join C *k;* then B C *k* will be the angle at which to cut the edge of the board at *a,* Fig. 2; and B C *f,* Fig. 1, the side of the board, *b a,* Fig. 2. H, Fig. 1, is the angle piece, and is found by making *o g* equal to *o f,* and joining D *g;* then the angle at H, will be the internal angle of the hopper box, and, if it is required to make the box at a greater or less angle than 45°, the angular piece at *h* will be, invariably, one half of the angle piece desired.

PLATE XXXV.

Shows the method of cutting luffer boards in a circular, or gothic-headed opening.

Fig. 1, represents an elevation of one half of an equilateral gothic-headed window; No. 1, No. 2, and No. 3, being boards, to be placed against the curve, as shown at Fig. 2.

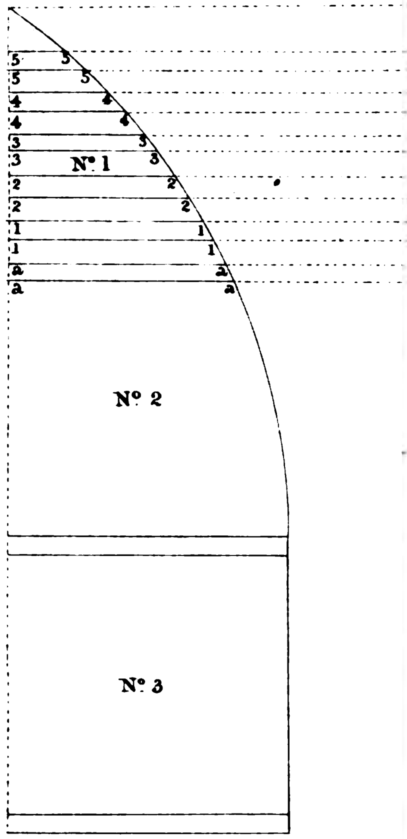
To find the points at which to cut the boards in order to fit against the circular jamb: Divide the board, No. 1, Fig. 2, into any number of equal parts, and from the points of division, 1, 2, 3, &c., on both sides, draw lines over the elevation of the board, parallel to its edges. With one point of the compass on *a,* describe the arcs, 1.1, 2.2, &c., making *a* 1, &c., on the line *a d* equal to *a* 1, &c., on the line *a b;* then *a d* will be the development of *a b.* Perpendicular to *a d,* and from the points, 1, 2, 3, &c., draw the parallel lines over the board to be cut at Fig. 3. Draw any line perpendicular to the edge of the board, Fig. 3, and transfer the distances between the vertical and curve lines, Fig. 1, to their corresponding line at Fig. 3. From the vertical line, and through the points thus found, trace the form of the boards required.

To make a perfect joint against the jamb, both sides will require to be found in the same manner.

Figure 4.

Is a plan of a circular seat.

Fig.1



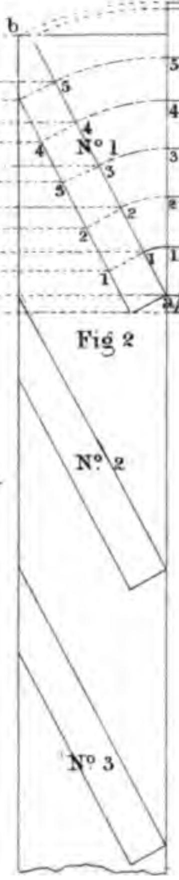
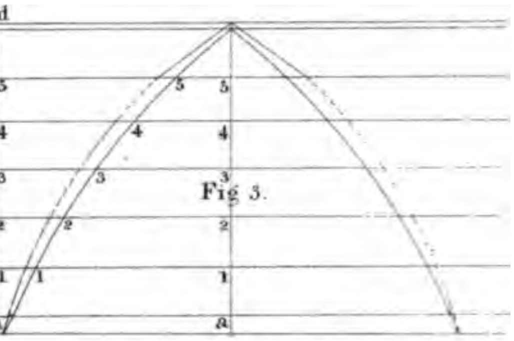


Fig 2



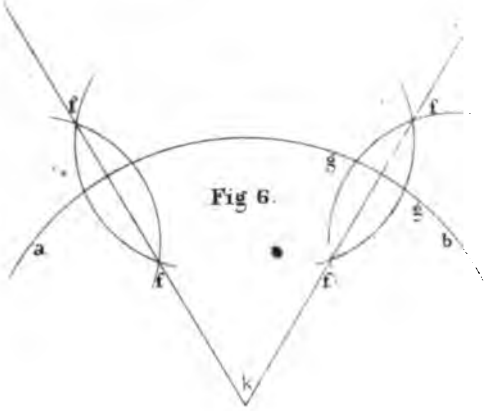


Fig 6.

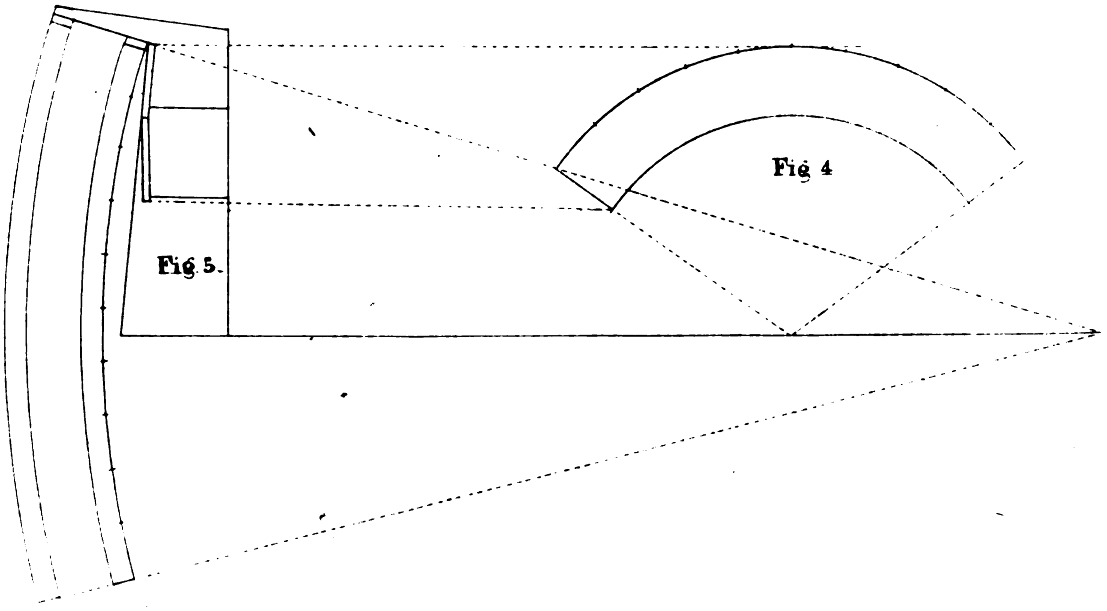


Fig 5.

Invented and Drawn by L.D. Gould.

HAND RAILING.

The art of forming Hand Rails round circular well-holes, without the use of a cylinder or mould, is a late invention.

The method of squaring the wreath, upon geometrical principles, was first published by Mr. Peter Nicholson, in 1792. The great attention to, and researches of, other eminent and skilful mechanics to this branch of building, have added much to the store of knowledge on this subject.

The methods here presented to the public, have been practised by the Author for many years, and are believed to be the least difficult to comprehend, of any now in use.

The Method of Drawing Scrolls for Hand Rails, answering to Every Description of Stairs.

Let E (Fig. 1, pl. 41,) be the center, EB the given radius, EA one-half the eye of the scroll, (which should be 3¼ or 4 inches diameter); divide AB in two equal parts in the point C; divide AC or CB into equal parts, consisting of one part more than the number of revolutions; in the line EB make EF and EI each equal to one-half of these parts; then, upon FI construct the square FGHI; draw GE and HE; divide GE and HE into as many equal parts as the spiral is to have revolutions. Through these points construct as many squares, of which one side will be in the line FI, and the other sides terminate in the lines GE and HE, then the angular points of the outer square will be the centers for the first revolution, and the angular points of the inner square will be the centers for the second revolution; the concave side of the rail is drawn from the same centers that the convex side is drawn from.

a n v, will contain the bevels for cutting the horizontal moulding at A. The bevel B, will give the face, and the bevel A, will give the butt-joint.

To find the face and butt-joint on the raking moulding.

Let X be the plan of the miter. Join *a s* and *b e*. At the section B, draw *b o*, perpendicular to *a b*. Draw *a o*, at right angles with *a s*. Make *a x* equal to *a b*. Draw *e i*, perpendicular to *b c*, and *x i* parallel to *b e*. Make *i q* equal to *b o*, and *q t* equal to *a t*, at X. Join I *t* and *i s;* then the angles *q i s* and *q i t*, will contain the bevels for cutting the face and butt-joints, for the raking moulding, at B. The bevel C, will give the face, and the bevel D, will give the butt-joint.

NOTE.—The method here shown will be found very convenient in mitering raking and horizontal mouldings, when they are sprung; and, as the surface of the moulding is irregular, it will be best to apply the bevels to a miter-box, taking care to place the mouldings in the box with the face side *a b*, on the bottom of the box.

PLATE XXXVII.

Figure 1.

Shows the method of finding the raking mould to any angle of elevation, also for a given elevation to any angle inclining towards a straight line.

Divide the surface of the horizontal moulding, here shown, into any number of parts, and from the points thus obtained, draw the inclining parallel lines to the required angle of elevation. From the same points draw vertical lines to meet the horizontal line *a b*, and, with one point of the compass on *a*, transfer the points between *a* and *b*, on the horizontal line, to the inclining line; perpendicular to the inclining line, draw the corresponding lines to meet the parallels, and through the points of intersection trace the raking moulding required.

Figure 2.

Shows the method of finding the raking moulding to any angle inclining towards a straight line.

Let B A C be a right angle *a* A C a straight line, and A D B the elevation of the roof, to find the raking mould to the elevation A D B, that shall miter into the horizontal moulding A, Fig. 1, at the angle *a* A F. Produce D *g* to *y*, making *g y* equal to the curve line, *g y*. Join B *y;* make *g n* on the line *g y*, equal to that part of the curve line intercepted between *g* and *s*. Draw *u v*, parallel to *y* B, and V W perpendicular to A B; then the line A O B will be the angle of elevation at which to draw the raking moulding that shall stand over the plan, A *s* F, and miter with the horizontal moulding B, when elevated to the given height of the roof.

This method will apply to any angle, from a straight line to an angle of ninety degrees, and will be found very useful when the building has an obtuse and acute angle.

By inspection of the figure, it will be perceived that if the moulding were to continue in the straight line, though elevated, it would not require a different mould, but if that elevated line were to diverge from the straight, it would begin to form the right angle, and consequently would commence changing its form from the horizontal moulding to that of the raking one, as shown at Fig. 1. This method is designed to give the exact form, at any point between the right angle and the straight line.

Figure 3.

Shows the method of finding the angle bars of shop fronts. A bare inspection of the figure is sufficient to understand it.

PLATE XXXVIII.

SPRINGING AND BENDING MOULDINGS.

Figs. 1 and 2, are sections of what are termed, among carpenters and joiners, spring-mouldings; and the stuff from which they are obtained are thinner than if the angular piece were worked on the moulding. These mouldings require brackets, as at A A, placed at proper distances, either in a straight, or curved line. If they are curved, the moulding will require to be bent in the same manner as in covering the frustrum of a right curve.

Fig. 1.

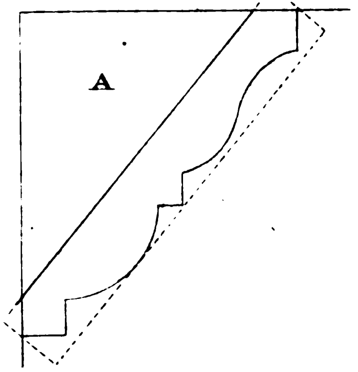


Fig. 2.

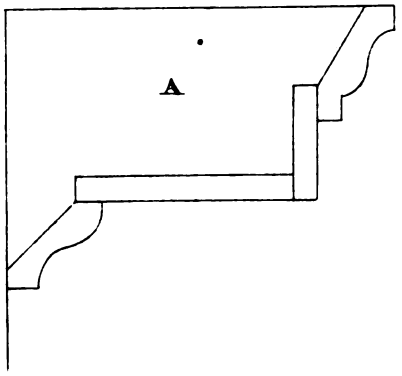


Fig. 3.

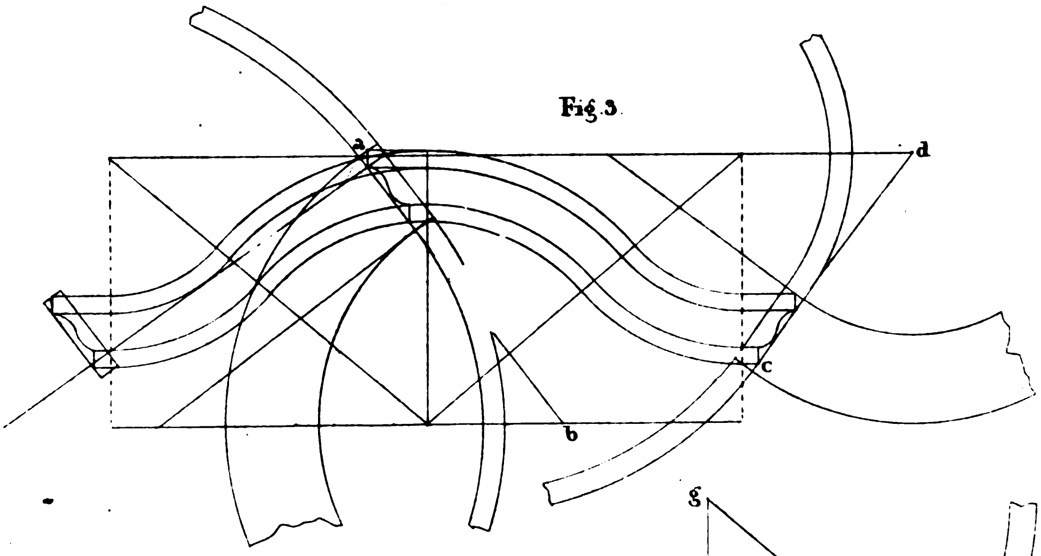


Fig. 5.

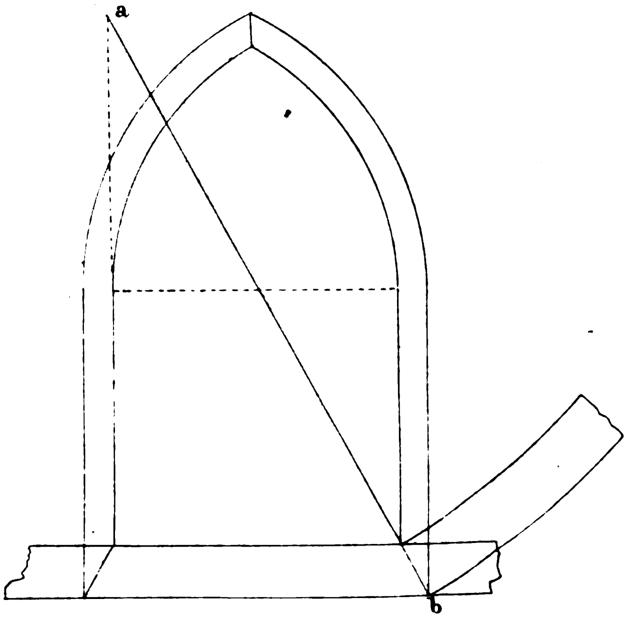
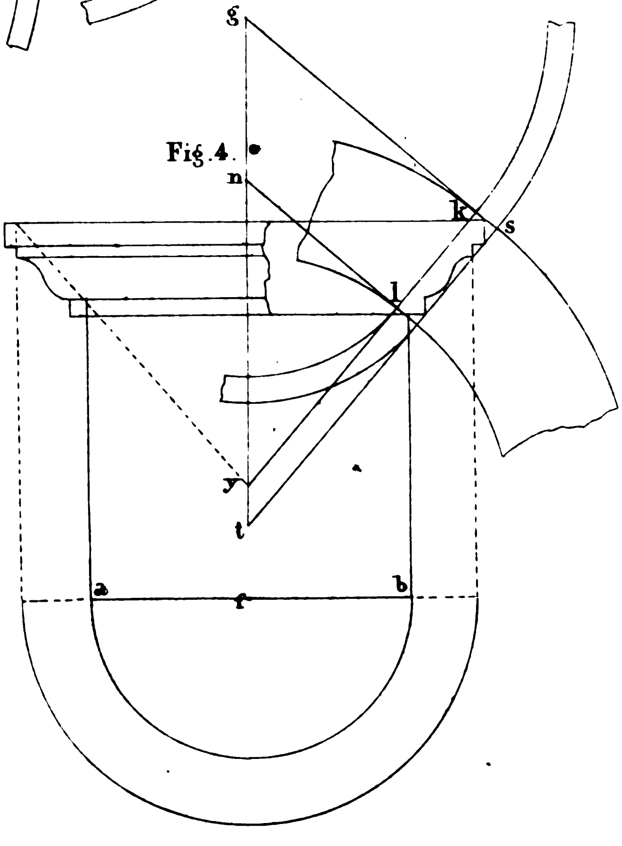


Fig. 4.



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to 1.1 2.2, &c., on the column; then through the points thus found, trace the curve on each side of the line ch, and you will have the form of the piece necessary to be removed, in order to allow the board to stand obliquely over the column. The same lines will be required for both sides of the board.

Figure 2.

Is the same thing applied to fitting a board against a cluster column.

Figure 3.

Is similar, showing the method of fitting a board over a diminished column. In this case, the cord of the circle, at each point of division, must be taken.

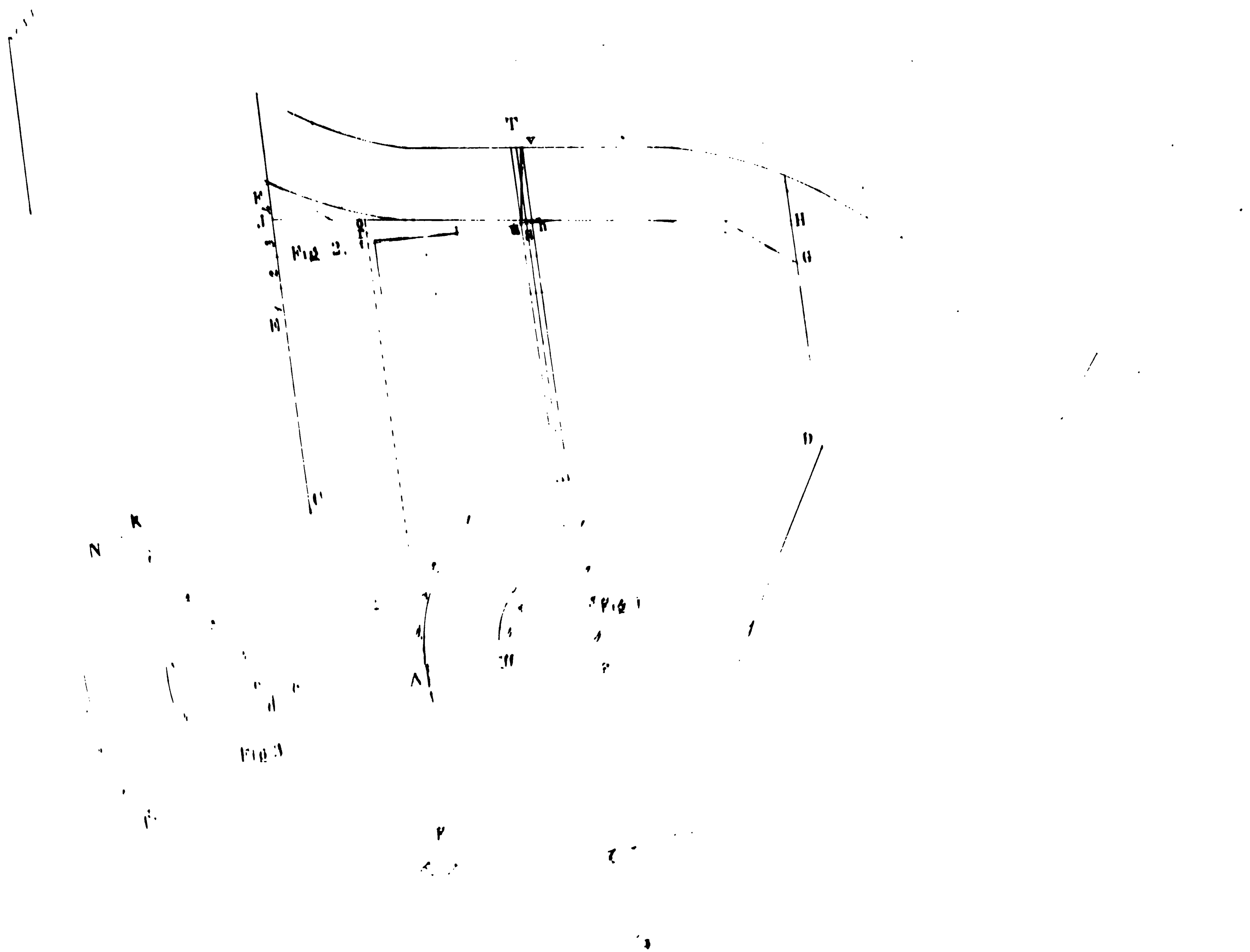


Fig 2.

Fig 1

Fig 3

The dotted squares show that one mould may be made to answer for any number of revolutions.

To find the face and falling moulds of the scroll.

Produce the line OB to R, and make RO equal to the width of the step; draw OT perpendicular to OR, and equal in height to the riser. Join TR; then through RB, raise the perpendiculars, 11.22, &c., to the line RT, and from the point of intersection on the line RT raise the perpendiculars RQ, 11.22, &c.; then with the compass transfer the distances on the plan 11.22, &c., (Fig. 1 to Fig. 2,) and through the points thus found trace the form of the face mould. In applying the face mould to the plank, the straight part of the rail should be parallel with the edge of the plank, as also in the remaining part of the scroll; the grain of the wood should be placed in the same direction, the pitch board will give the angle on the edge of the plank to apply the face mould, so that if the plank were placed in the position of the rail when finished, the mould would be in the same vertical line.

To draw the falling mould, make the line TO, (figure 3,) equal to the stretch-out of the convex side of the scroll, (figure 1,) from O around to T. Raise the perpendicular OT equal to the height of the rise; make OR equal to the width of the step; join RT, which will be the rake of the stairs; divide AB and BC each into the same number of equal parts, as in this case, 9, then with a straight edge draw the intersecting line, which will form the upper edge of the falling mould required.

In applying the falling mould, the point C, (figure 3,) should be placed on the point T, (figure 2.)

Plate 42 *shows an easy method of obtaining the moulds for platform stairs, adapted to small openings, from six to twelve inches in diameter.*

To get the Falling Mould,—Let Figure 1 be the plan of the rail, and *co* the stretch-out of the convex side of the rail; EF the height of the rise; divide the rise in four equal parts, make EJ equal to three parts, and GH equal to one part. Join HJ, place the pitch board at G and F, and produce the raking lines thus found to the line HJ; then with the compass describe the easings which complete the under side of the falling moulds. To cut the butt joint, draw the thickness of the rail parallel to HJ, divide ST in two equal parts, and through the center and at right angles to HJ draw *uv*; then from the points *uv* draw the lines *u*X and *v*Y; produce FA, figure 1, to *o*, figure 2; draw *ht* parallel to EG; divide *to* in two equal parts, then *trh* will be the angle to draw the face mould, and the angle to set the bevel for the edge of the plank before applying the face mould.

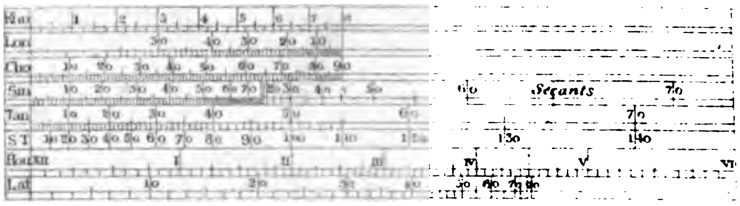
To draw the face mould, (figure 2,) let the line FK, (figure 3,) be the rake or pitch of the stairs; draw *cdf* at right angles to FK, make *cd* equal to *tr*, (figure 2,) and through the point *d*, draw TN parallel to FK; produce A*c* to *e*, make *df* equal to *rh*, (figure 2,) and join *ef*, which is the ordinate line for the face mould; take in the compasses the distance *ef*, and with one point on A, (figure 1,) describe the arc cutting the line *yh* in *p*. Join A*p*, then A*p* is the ordinate line for the plan; draw any number of lines parallel to A*c* and CK, and from their intersection with the convex side of the plan and the line TN, draw the ordinates parallel to A*p* and *ef*; then transfer the distances from figure 1 to figure 3, making 11.22, &c., equal to 11.22, &c., and through the points thus found trace the form of the face mould required.

Plate 43 shows a continuation of the rail around the 2d story cylinder. Figure 1 shows the easing from the short flight to the 2d floor; figure 2 the falling mould for the 2d flight of stairs. The student will observe, that in order to give the same length of balluster on the level that is required on the center of the step, we raise half the height of one rise at figure 1. Also, that the face mould drawn for the 2d story platform will answer for figure 2, by planing off the overwood shown at *sh*, (figure 2, plate 42.) Figure 3 shows the method of finding the point to bore for the first balluster on the 2d step; when you have the length of the newel, and the length of the short balluster given, let A be the pitch board, and *c* the center of the newel post. Make *de* equal to the difference between the lengths of the newel post and the short balluster, and at right angles to *de* draw *ef*; place the line *ef* on the line with the underside of the newel cap, and where the line *de* intersects, the underside of the rail will be the point for the first short balluster. Figure 3, plate 40, shows the method of mitering the rail in the newel cap. This method of getting out a hand rail for small openings, is believed to be the best now in use, inasmuch as it requires fewer joints and avoids crossing the grain of the wood, and is obtained of the least possible thickness of plank.

Plate 44 shows the method of finding the moulds for executing a hand rail, over a semi-circular plan of five winders, out of the least possible thickness of plank.

Let figure 1 be the plan of the rail, AB the stretchout of the convex side, and A*c* the height of six rises, place the pitch board KK, as shown, at *d* and *e*. Join *d* and *c*, and complete the falling mould by easing the angles *c* and *d* either by intersecting lines or with the compass. It will be observed here, that by increasing the width of the first and last winder, that they will follow the line of the easing, and give a more graceful and easy curve to the string piece. As it is customary to attach a piece of straight rail to the circular part, for the convenience of joining, the length of which is usually three or four inches—therefore, make AK and BR equal to the straight part of the rail; draw R*s* and K*s* perpendicular to KR, cutting the upper edge of the falling mould in *ss* on the plan figure 1, draw the chord lines 11.22, and from the point *o* draw the radiating lines *op* and *ot* at right angles to the chord lines 11.22, make *ru* equal to the stretchout of *tp*, and draw *rs* and *us* perpendicular to *ru*, cutting the upper edge of the falling mould in the points *ss;* the butt joints are drawn at right angles to the edge of the falling mould, as shown at *sws*. To draw the face mould for the lower quarter of the circle—Let figure 2 be the plan, *rt* the chord line; from the point *o* draw *o*W perpendicular to the chord line *rt*, draw *ts* and *rs* parallel to Y*w*, make *ts* equal to R*s*, and *rs* equal Q*s*, figure 1, and through the points *ss* draw the line *sws;* make *us* on the line Y*w* equal *us*, figure 1; make *yh* equal to *sv*, and *vn* equal to *he*, then *wn* will be the ordinate for the face mould. To find the ordinate for the plan, make *ez* equal to *wn*, draw any number of lines parallel to *rs*, and from their intersection with the line *ss*, and the convex side of the plan, draw their ordinates parallel to *ez* and *wn*, and transfer the distances 11.22, &c., and through the points thus found trace the form of the face mould required. Figure 3 shows the upper rail piece; the heights are taken from the line CX, and is obtained in the same way as the lower rail piece. The spring bevel is found by making 12 at HH, each equal to *sv* or *yh*, figure 2 and figure 3. The down or plumb bevels are shown at numbers 1, 2, 3, and 4.

To demonstrate the principle upon which this method is based, we will suppose the line KR to be placed over the convex side of the plan, and the perpendiculars R*s*, *us*, &c., to stand over the points *pqt*, &c. Then suppose a line to be drawn from *s* to *s*, over 11 and 22, and parallel to the chord lines 11 and 22. Then the plane from the point *s* to the chord line will be oblique to the base, and form an acute angle, as shown at H, figure 2, and the opposite side will form an obtuse angle, as shown at H, figure 3. Then suppose the face mould, figure 2, to rest on the points *ssw*, in the point *m* and *ss*, which will be the position to require the least possible thickness of plank to form the rail.



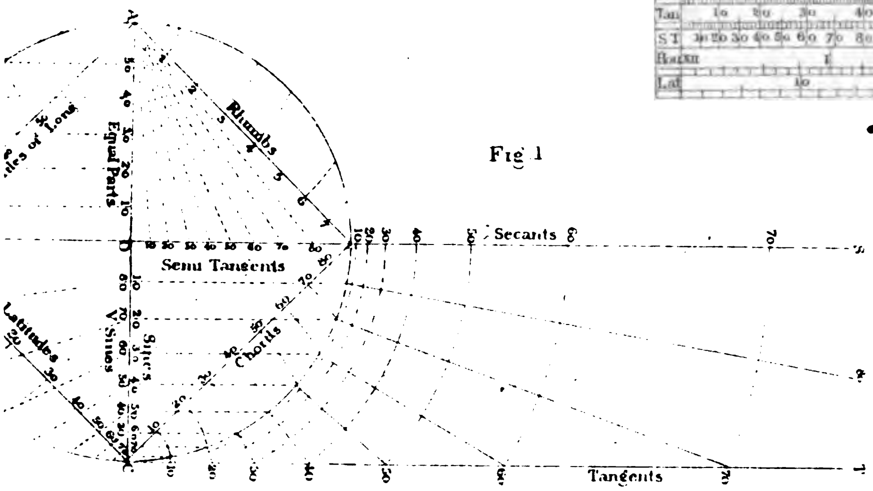


Fig 1

Fig. 4

Fig 2

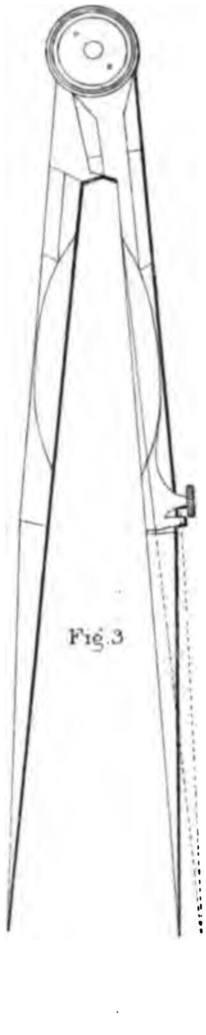


Fig. 3

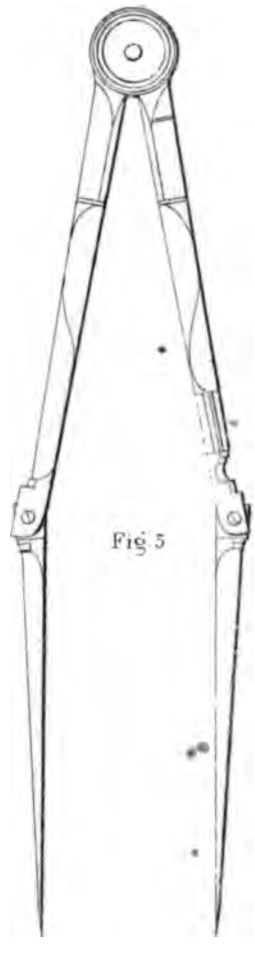


Fig 5

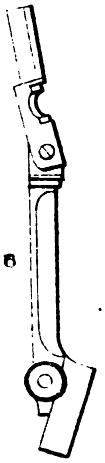


Fig 6

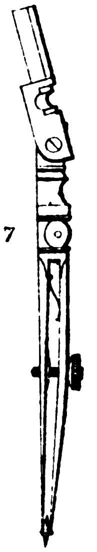


Fig 7

Fig 8

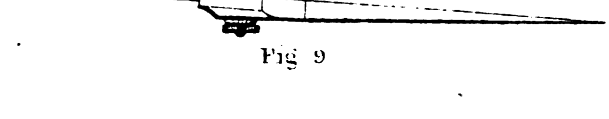


Fig 9

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On Drawing Compasses.—This instrument consists of two legs movable about a joint, so that the points at the extremities of the legs may be set at any required distance from one another; it is used to transfer and measure distances, and to describe arcs and circles.

The points of the compasses should be formed of well-tempered steel, that cannot easily be bent or blunted, the upper part being formed of brass or silver. The joint is framed of two substances; one side being of the same material as the upper part of the compasses, either brass or silver, and the other of steel. This arrangement diminishes the wear of the parts, and promotes uniformity in their motion. If this uniformity be wanting, it is extremely difficult to set the compasses at any desired distance, for, being opened or closed by the pressure of the finger, if the joint be not good, they will move by fits and starts, and either stop short of, or go beyond the distance required; but, when they move evenly, the pressure may be regulated so as to open the legs to the desired extent, and the joint should be stiff enough to hold them in this position, and not to permit them to deviate from it in consequence of the small amount of pressure which is inseparable from their use. When greater accuracy in the set of the compasses is required than can be effected by the joint alone, we have recourse to the

Hair Compasses, in which the upper part of one of the steel points is formed into a bent spring, which, being fastened at one extremity to the leg of the compasses almost close up to the joint, is held at the other end by a screw. A groove is formed in the shank, which receives the spring when screwed up tight; and, by turning the screw backwards, the steel point may be gradually allowed to be pulled backwards by the spring, and may again be gradually pulled forwards by the screw being turned forwards.

Fig. 2 represents these compasses when shut; Fig. 3 represents them open, with the screw turned backwards, and the steel point p, in consequence moved backwards by its spring s, from the position represented by the dotted lines, which it would have when screwed tight up.

Fig. 4 represents a key, of which the two points fit into the two holes seen in the nut n, of the joint; and by turning this nut the joint is made stiffer or easier at pleasure.

To take a Distance with the Hair Compasses.—Open them as nearly as you can to the required distance, set the fixed leg on the point from which the distance is to be taken, and make the extremity of the other leg coincide accurately with the end of the required distance, by turning the screw.

COMPASSES WITH MOVABLE POINTS.—If an arc or circle is to be described faintly, merely as a guide for the terminating points of other lines, the steel points are generally sufficient for the purpose, and are susceptible of adjustment with greater accuracy than a pencil point; but, in order to draw arcs or circles with ink or black lead, compasses with a movable point are used. In the best description of these compasses the end of the shank is formed into a strong spring, which holds firmly the movable point, or a pencil or ink point, as may be required. A lengthening bar may also be attached between the shank and the movable point, so as to strike larger circles, and measure greater distances. The movable point to be attached to the lengthening bar, as also the pen point and pencil point, are furnished with a joint, that they may be set nearly perpendicular to the paper.

Fig. 5, the compasses, with a movable point at B.

C and D, the joints to set each point perpendicular to the paper.

Fig. 6, the pencil point.

Fig. 7, the pen point. (This is represented with a dotting wheel, the pen point and the dotting point being similar in shape to each other.)

Fig. 6, the lengthening bar.

To describe small arcs or circles, a small pair of compasses, called *bow compasses*, with a permanent ink or pencil point, are used. They are formed with a round head, which rolls with ease between the fingers. Figs. 9 and 10 represent two constructions of pen bows. Fig. 9 to describe arcs of small radii with exactness, by means of the adjusting screw, C.

For copying and reducing drawings, compasses of a peculiar construction are used; the simplest form of which is that called wholes and halves, (Fig. 11, Plate 46,) because the longer legs being twice the length of the shorter, when the former are opened to any given line, the shorter ones will be opened to the half of that line. By their means, then, all the lines of a drawing may be reduced to one half, or enlarged to double their length. These compasses are also useful for dividing lines by continual bisections.

PROPORTIONAL COMPASSES.—By means of this ingenious instrument drawings may be reduced or enlarged, so that all the lines of the copy, or the areas of solids represented by its several parts, shall bear any required proportion of the lines, areas, or solids of the original drawing. They will also serve to inscribe regular polygons in circles, and to take the square roots and cube roots of numbers. In Fig. 12 the scale of lines is placed on the leg, A E, on the left-hand side of the groove, and the scale of circles, on the same leg, on the right-hand side of the groove. The scales of plans and solids are on the other face of the instrument.

To set the instrument it must first be accurately closed, so that the two legs appear but as one; the nut C, being then unscrewed, the slider may be moved, until the line across it coincide with any required division upon any one of the scales. Now tighten the screw, and the compasses are set.

To reduce or enlarge the Lines of a Drawing.—The line across the slider being set to one of the divisions, 2, 3, 4, &c., on the scale of lines, the points A B, will open to double, triple, four times, &c., the distances of the points D E. If, then, the points A and B be opened to the lengths of the lines upon a drawing, the points D and E will prick off a copy with the lines reduced in the proportions of $\frac{1}{2}$ to 1, $\frac{1}{3}$ to 1, $\frac{1}{4}$ to 1, &c.; but, if the points D and E be opened to the lengths of the lines upon a drawing, the points A and B will prick off a copy with the lines enlarged in the proportions of 2 to 1, 3 to 1, 4 to 1, &c.

To inscribe in a Circle a regular Polygon of any Number of Sides from 6 *to* 20.—The line across the slider being set to any number on the scale of circles, and the points A and B being opened to the length of any radius, the points D and E will prick off a polygon on that number of sides, in the circle described with this radius; thus, if the line across the slider be set to the division marked 12 on the scale of circles, and a circle be described with the radius A B, D E will be the cord of a $\frac{1}{12}$th part of the circumference, and will prick off a regular polygon of 12 sides in it.

To reduce or enlarge the Area of a Drawing.—The numbers upon the scale of plans are the squares of the ratios of the lengths of the opposite ends of the compasses, when the line across the slider is set to those numbers; and, the distances between the points being in the same ratio as the lengths of the corresponding ends, the areas of the drawings, and of the several parts of the drawings, pricked off by these points, will have to one another the ratio of 1 to the number upon the scale of plans to which the instrument is set. Thus, if the line across the slider be set to 4 on the scale of plans, the distance between the points A and B will be twice as great as the distance between D and E; and, if A and B be opened out to the lengths of the several lines by a drawing, D and E will prick off a copy occupying ¼th the area; if the line across the slides be set to 5 on the same scale, the distances between the points will be in the ratio of 1 to $\sqrt{5}$, and the area of the copy pricked off by the points D and E will be the $\frac{1}{5}$th of the area of the drawing, of which the lines are taken off by A and B: conversely, if the lines of the drawing be taken off by the points D and E, the points A and B will prick off a copy, of which the area will be 4 times or 5 times as great, according as the line across the slider is set to the division marked 4 or 5 on the scale.

To take the Square Root of a Number.—The line across the slider being set to the number upon the scale of plans, open the points A and B to take the number from any scale of equal parts, then the points D and E applied to the same scale of equal parts will take the square root of the number. Thus, to take the square root of 3, set the line across the slider to 3, open out the compass, till A and B take off 3 from any scale of equal parts, and the points D and E will take off 1.73, which is the square root of 3, from the same scale of equal parts. A mean proportional between two numbers, being the square root of their product, may be found by multiplying the numbers together, and then taking the square root of the product in the manner explained above.

The numbers of the scale of solids are the cubes of the ratios of the lengths of the opposite ends of the compasses, when the line across the slider is set to those numbers; so that, when this line is set to the division marked 2 upon the scale of solids, the distance between the points A and B will give the side of a solid of double the content of that, of which a like side is given by the distance of the points D and E, when the line is set to 3, the respective distances of the points will give the like sides of solids, the contents of which will be in the proportion of 3 to 1; and so on.

The Cube Root of a given number may be found by setting the line across the slider to the number upon the scale of solids, and, opening the points A B, to take off the number upon any scale of equal parts, the points D E, will then take off the required cube root from the same scale.

The Triangular Compasses.—One of the best forms of these instruments is represented in Fig. 13. *a b c* is a solid tripod, having at the extremity of the three arms three limbs, *d*, *e*, and *f*, moving freely upon centers by which they may be placed in any position with respect to the tripod and each other. These limbs carry points at right angles to the plane of the instrument, which may be brought to coincide, in the first instance, with any three points on the original, and then transferred to the copy. After this first step two of these points must be set upon two points of the drawing already copied, and the third made to coincide with a new point of the drawing, that is, one not yet copied: then, by placing the two first points on the corresponding points in the copy, the third point of the compasses will transfer the new point to the copy.

Another form of triangular compasses is represented at Fig. 14.

The Drawing-Pen.—This instrument is used for drawing straight lines. It consists of two blades with steel points fixed to a handle; and they are so bent, that a sufficient cavity is left between them for the ink, when the ends of the steel points meet close together, or nearly so. The blades are set with the points more or less open by means of a mill-headed screw, so as to draw lines of any required fineness or thickness. One of the blades is framed with a joint, so that by taking out the screw the blades may be completely opened, and the points effectively cleaned after use. The ink is to be put between the blades by a common pen, and in using the pen it should be slightly inclined in the direction of the line to be drawn, and care should be taken that both points touch the paper; and these observations equally apply to the pen points of the compasses before described. The drawing pen should be kept close to the straight edge, and in the same direction during the whole operation of drawing the line.

For drawing close parallel lines in mechanical and architectural drawings, or to represent canals or roads, a double pen (Fig. 16) is frequently used, with an adjusting screw to set the pen to any required small distance. This is usually called the road pen. The best pricking point is a fine needle held in a pair of forceps (Fig. 17). It is used to mark the intersections of lines, or to set off divisions from the plotting scale and protractor. This point may also be used to prick through a drawing upon an intended copy, or, the needle being reversed, the eye end forms a good tracing point.

A Straight Edge.—As many instruments are required to have straight edges for the purpose of measuring distances, and of drawing straight lines, it may be considered important to test the accuracy of such edges. This may be done by placing two such edges in contact and sliding them along each other, while held up between the eye and the light: if the edges fit close in some parts, so as to exclude the light, but permit it to pass between them at other parts, the edges are not true: if, however, the edges appear, as far as the test has now proceeded, to be true, still this may arise from a curvature in one edge fitting into an opposite curvature in the other; the final step then is to take a third edge, and try it in the same manner with each of the other two, and, if in each case the contact be close throughout the whole extent of the edges, then they are all three good.

To draw a straight line between two points upon a plane, we lay a rule so that the straight edge thereof may just pass by the two points; then, moving a fine pointed needle, or drawing pen, along this edge, we draw a line from one point to the other, which, for common purposes, is sufficiently exact; but, where great accuracy is required, it will be found extremely difficult to lay the rule equally with respect to both the points, so as not to be nearer to one point than the other. It is difficult also so to carry the needle, or pen, that it shall neither incline more to one side than the other of the rule; and, thirdly, it is very difficult to find a rule that shall be perfectly straight.

"If the two points be very far distant, it is almost impossible to draw the line with accuracy and exactness; a circular line may be described more easily, and more exactly, than a straight or any other line, though even then many difficulties occur, when the circle is required to be of a large radius.

"And let no one consider these reflections as the effect of too scrupulous exactness, or as an unnecessary aim at precision; for, as the foundation of all our knowledge in geography, navigation, and astronomy, is built on observations, and all observations are made with instruments, it follows that the truth of the observations and the accuracy of the deductions therefrom, will principally depend on the exactness with which the instruments are made and divided, and that those sciences will advance in proportion as these are less difficult in their use, and more perfect in the performance of their respective operations.

On Scales.--Scales of equal parts are used for measuring straight lines, and laying down distances, each part answering for one foot, one yard, one chain, &c., as may be convenient, and the plan will be larger or smaller as the scale contains a smaller or a greater number of parts in an inch.

Scales of equal parts may be divided into three kinds; simply divided scales, diagonal scales, and vernier scales.

Simply divided Scales.—Simply divided scales consist of any extent of equal divisions, which are numbered 1, 2, 3, &c., beginning from the second division on the left hand. The first of these primary divisions is subdivided into ten equal parts, and from these last divisions the scale is named. Thus it is called a scale of 30, when 30 of these small parts are equal to one inch. If, then, these subdivisions be taken as units, each to represent one mile, for instance, or one chain, or one foot, &c., the primary divisions will be so many tens of miles, or of chains, or of feet, &c.; if the subdivisions are taken as tens, the primary divisions will be hundreds; and, if the primary divisions be units, the subdivisions will be tenths.

Fig. 18 represents six of the simply divided scales, which are generally placed upon the plane scale. To adapt them to feet and inches, the first primary division is divided duodecimally upon an upper line. To lay down 360, or 36, or 3.6, &c., from any one of these scales, extend the compasses from the primary division numbered 3 to the 6th lower subdivision, reckoning backwards, or towards the left hand. To take off any number of feet and inches, 6 feet 7 inches for instance, extend the compasses from the primary division numbered 6, to the 7th upper subdivision, reckoning backwards, as before.

Diagonal Scales.—In the simply divided scales one of the primary divisions is subdivided only into ten equal parts, and the parts of any distance which are less than tenths of a primary division cannot be accurately taken off from them; but, by means of a diagonal scale, the parts of any distance which are the hundredths of the primary divisions are correctly indicated, as will easily be understood from its construction, which we proceed to describe.

Draw eleven parallel equidistant lines; divide the upper of these lines into equal parts of the intended length of the primary divisions; and through each of these divisions draw perpendicular lines, cutting all the eleven parallels, and number these primary divisions, 1, 2, 3, &c., beginning from the second.

Subdivide the first of these primary divisions into ten equal parts, both upon the highest and lowest of the eleven parallel lines, and let these subdivisions be reckoned in the opposite direction to the primary divisions, as in the simply divided scales.

Draw the diagonal lines from the tenth subdivision below to the ninth above; from the ninth below to the eighth above: and so on; till we come to a line from the first below to the zero point above. Then, since these diagonal lines are all parallel, and consequently everywhere equidistant, the distance between any two of them in succession, measured upon any of the eleven parallel lines which they intersect, is the same as this distance measured upon the highest or lowest of these lines, that is, as one of the subdivisions before mentioned: but the distance between the perpendicular, which passes through the zero point, and the diagonal through the same point, being nothing on the highest line, and equal to one of the subdivisions on the lowest line, is equal to one-tenth of a subdivision on the second line, to two-tenths of a subdivision on the third, and so on; so that this, and consequently each of the other diagonal lines, as it reaches each successive parallel, separates further from the perpendicular through the zero point by one-tenth of the extent of a subdivision, or one-hundredth of the extent of a primary division. Fig. 19 represents the two diagonal scales which are usually placed upon the plane scale of six inches in length. In one, the distances between the primary divisions are each half an inch, and in the other a quarter of an inch. The parallel next to the figures numbering these divisions must be considered the highest or first parallel in each of these scales, to accord with the above description.

The primary divisions being taken for units, to set off the numbers 5·74 by the diagonal scale. Set one foot of the compasses on the point where the fifth parallel cuts the eighth diagonal line, and extend the other foot to the point where the same parallel cuts the sixth vertical line.

The primary divisions being reckoned as tens, take off the number 467. Extend the compasses from the point where the eighth parallel cuts the seventh diagonal to the point where it cuts the fifth vertical.

The primary divisions being hundreds, to take off the number 25·3. Extend the compasses from the point where the fourth parallel cuts the sixth diagonal to the point where it cuts the third vertical.

Now, since the first of the parallels, of the diagonals, and of the verticals indicate the zero points for the third, second, and first figures respectively, the second of each of them stands for, and is marked 1, the third 2, and so on, and we have the following

General Rule.—To take off any number to three places of figures upon a diagonal scale. On the parallel indicated by the third figure, measure from the diagonal indicated by the second figure to the vertical indicated by the first.

Vernier Scales.—The nature of these scales will be understood from their construction. To construct a vernier scale, which shall enable us to take off a number to three places of figures: divide all the primary divisions into tenths, and number these subdivisions 1, 2, 3, &c., from the left hand towards the right throughout the whole extent of the scale.

Take off now, with the compasses, eleven of these subdivisions, set the extent off backwards from the end of the first primary division, and it will reach beyond the beginning of this division, or zero point, a distance equal to one of the subdivisions. Now divide the extent thus set off into ten equal parts, marking the divisions on the opposite side of the divided line to the strokes marking the primary divisions and the subdivisions, and number them 1, 2, 3, &c., backwards from right to left. Then, since the extent of eleven subdivisions has been divided into ten equal parts, so that these ten parts exceed by one subdivision the extent of ten subdivisions, each one of these equal parts, or, as it may be called, one division of the vernier scale, exceeds one of the subdivisions by a tenth part of a subdivision, or a hundredth part of a primary division. In our figure the distances between the primary divisions are each one inch, and consequently, the distances between the subdivisions are each one tenth of an inch, and the distances between the divisions of the vernier scale each one tenth and one hundredth of an inch.

To take off the number 253 from the scale, Fig. 20. Increase the first figure 2 by 1, making it 3; because the vernier scale commences at the end of the first primary division, and the primary divisions are measured from this point, and not from the zero point.* The first thus increased with the second now represents 35 of the subdivisions from the zero point, from which the third figure 3, must be subtracted, leaving 32; since three divisions of the vernier scale will contain three of these subdivisions, together with three-tenths of a subdivision. Place, then, one point of the compasses upon the third division of the vernier scale, and extend the other point to the 32d subdivision, or the second division beyond the 3d primary division, and, laying down the distance between the points of the compass, it will represent 253, or 25.3, or 2·53, according as the primary divisions are taken as hundreds, tens, or units.

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* If the vernier scale were placed to the left of the zero point, a distance less than one primary division could not always be found upon the scale.

General Rule.—To take off any number to three places of figures upon this vernier scale. Increase the first figure by one; subtract the third figure from the second, borrowing one from the first increased figure, if necessary, and extend the compasses from the division upon the vernier scale, indicated by the third figure, to the subdivision indicated by the number remaining after performing the above subtraction.

Suppose it were required to take off the number 253·5. By extending the compasses from the 3d division of the vernier scale to the 32d subdivision, the number 253 is taken off, as we have seen. To take off, therefore, 253·5, the compasses must be extended from one of these points to a short distance beyond the other. Again, by extending the compasses from the 4th division of the vernier scale to the 31st subdivision, the number 254 would be taken off. To take off 253.5, then the compasses must be extended from one of these points to within a short distance of the other; and, by setting the compasses so that, when one point of the compasses is set successively on the 3d and 4th division of the vernier scale, the other point reaches as far beyond the 32d subdivision as it falls short of the 31st, the number 253·5 is taken off. If the excess in one case be twice as great as the defect in the other, the distance represents the number 25·3⅔, or 253·66; and if the excess be half the defect, the distance represents 253⅓, or 253·33. Thus distances may be set off with an accurately constructed scale of this kind to within the three-hundredth of a primary division, unless these divisions be themselves very small.

We are not aware that a scale of this kind has been put upon the plain scales sold by any of the instrument makers; but, during the time occupied in plotting an extensive survey, the paper which receives the work is affected by the changes which take place in the hygrometrical state of the air, and the parts laid down from the same scale, at different times, will not exactly correspond, unless this scale has been first laid down upon the paper itself, and all the divisions have been taken from the scale so laid down, which is always in the same state of expansion as the plot. For plotting, then, an extensive survey, and accurately filling in the minutæ, a diagonal, or vernier scale may advantageously be laid down upon the paper upon which the plot is to be made. A vernier scale is preferable to a diagonal scale, because in the latter it is extremely difficult to draw the diagonals with accuracy, and we have no check upon its errors; while in the former the uniform manner in which the strokes of one scale separate from those of the other is some evidence of the truth of both.*

ON THE PROTRACTING SCALES.—The nature of these scales will be understood from the following construction (Plate I. Fig. 1.)

With centre O, and radius O A, describe the circle A B C D; and through the centre O, draw the diameters A C, and B D, at right angles to each other, which will divide the circle into four quadrants, A B, B C, C D, and D A.

Divide the quadrant C D, into nine equal parts, each of which will contain ten degrees, and these parts may again be subdivided into degrees, and, if the circle be sufficiently large, into minutes.

Set one foot of the compasses upon C, and transfer the divisions in the quadrant C D, to the right line C D, and we shall have a scale of chords.†

From the divisions in the quadrant C D, draw right lines parallel to D O, to cut the radius O C, and numbering the divisions from O, towards C, we shall have a scale of sines.

If the same divisions be numbered from C, and continued to A, we shall have a scale of versed sines.

From the centre O, draw right lines through the divisions of the quadrant C D, to meet the line C T, touching the circle at C, and numbering from C, towards T, we shall have a scale of tangents.

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* In Mr. Bird's celebrated scale, by means of which he succeeded in dividing, with greatly improved accuracy, the circles of astronomical instruments, the inches are divided into tenths, as in the scale described in the text, and 100 of these tenths are divided into 100 parts for the vernier scale.

† We give the constructions in the text to show the nature of the scales; but in practice a scale of chords is most accurately constructed by values computed from tabulated arithmetical values of sines, which computed values are set off from a scale of equal parts; and the circle is divided most accurately by means of such computed chords. The limits of our work forbid our entering further upon this subject. All the other scales will also be most accurately constructed from computed arithmetical values, taken off by means of the beam compasses hereafter described, and corrected by the aid of Bird's vernier scale.

Set one foot of the compass upon the center O, and transfer the divisions in C T into the right line O S, and we shall have a scale of secants.

Right lines, drawn from A to the several divisions in the quadrant C D, will divide the radius O D into a line of semi-tangents, or tangents of half the angles indicated by the numbers; and the scale may be continued by continuing the divisions from the quadrant C D, through the quadrant D A, and drawing right lines from A, through these divisions to meet the radius O D, produced.

Divide the quadrant A D into eight equal parts, subdivide each of these into four equal parts, and, setting one foot of the compasses upon A, transfer these divisions to the right line A D, and we shall have a scale of rhumbs.

Divide the radius A O into 60 equal parts, and number them from O towards A; through these divisions draw right lines parallel to the radius O B, to meet the quadrant A B; and, with one foot of the compasses upon A, transfer these divisions from the quadrant to the right line A B, and we shall have a scale of longitudes.

Place the chord of 60°, or radius,* between the radii O C and O B, meeting them at equal distances from the center; divide the quadrant C B into six equal parts, for intervals of hours, subdividing each of these parts into 12 for intervals of 5 minutes, and further subdividing for single minutes if the circle be large enough; and from the center O draw right lines to the divisions and subdivisions of the quadrant, intersecting the chord or radius placed in the quadrant; and we shall have a scale of hours.

Prolong the touching line T C to L; set off the scale of sines from C to L; draw right lines from the center O to the divisions upon C L, and from the intersections of these lines with the quadrant C B draw right lines parallel to the radius O C, to meet the radius O B, and we shall have a scale of latitudes.†

Corresponding lines of hours and latitudes may also be constructed (as represented in our figure) more simply, and on a scale twice as large as by the preceding method, as follows:

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* Chord of 60° is equal to radius. Euc. book iv. prop. 15. Cor.

† The line of latitudes is a line of sines, to radius equal to the whole length of the line of hours, of the angles, of which the tangents are equal to the sines of the latitudes. The middle of the hour line being numbered for three o'clock, the divisions for the other hours are found by setting off both ways from the middle the tangents of n. 15°, n. being the number of hours from three o'clock, that is, one for two o'clock and four o'clock, two for one o'clock and five o'clock, and three for twelve o'clock and six o'clock.

With the chord of 45° set off from B to E, and again from B to F, we obtain a quadrant E F bisected in B; and, the chord of 60° or radius being set off from A, C, F, and E, this quadrant is divided into six equal parts. From the center O, draw straight lines through these divisions to meet the line touching the circle at B, and we shall have the line of hours.

From the point D, draw right lines through the divisions upon the line of sines O C, to meet the circumference B C, and transferring these divisions from B, as a center to the chord B C, we shall have the corresponding line of latitudes.

It is not necessary that these scales should all be projected to the same radius; but those which are used together, as the rhumbs and chords, the chords and longitudes, the sines, tangents, secants, and semitangents, and, lastly, the hours and latitudes, must be so constructed necessarily. In the accompanying diagram (plate 1, fig. 2) we have laid down the hours and latitudes to a radius equal to the whole length of the scale, the other lines being laid down to the radius used in the foregoing construction.

The Line of Chords is used to set off an angle, or to measure an angle already laid down.

1st. *To set off an angle*, which shall contain D° from the point A, in the straight line A B. Open the compasses to the extent of 60° upon the line of chords, which equals the radius to which this line has been laid down, and, setting one foot upon A, Fig. 21, with this extent describe an arc cutting A B in B; then, taking the extent of D° from the same line of chords, set it off from B to C; and, joining A C, B A C is the angle required. Thus to set off an angle of 41°, having described the arc B C, as directed, with one foot of the compasses on B, and the extent of 41° on the line of chords, intersect B C in C, and join A C.

2d. To measure the angle contained by the straight lines A B and A C, already laid down. Open the compasses to the extent of 60° on the line of chords, as before, and with this radius describe the arc B C, cutting A B and A C, produced, if necessary, in the points B and C; then, extending the compasses from B to C, place one point of the compasses on the beginning, or zero point, of the line of chords, and the other point will extend to the number upon this line, indicating the degrees in the angle BAC. If, for instance, this point fall on the 41st division, or the first division beyond that marked ·40 in the figure (plate 1, fig. 2), the angle BAC will contain 41°.

The Line of Rhumbs is a scale of the chords of the angles of deviation from the meridian denoted by the several points and quarter points of the compass, enabling the navigator, without computation, to lay down or measure a ship's course upon a chart. Thus, supposing the ship's course to be N.N.E. $\frac{3}{4}$ E. Through the point A, fig. 22, representing the ship's place upon the chart, draw the meridian AB, and with center A and distance equal to the extent of 60° upon the line of chords describe an arc cutting AB in B; then on the line of rhumbs take the extent to the third subdivision beyond division marked 2, because N.N.E. is the second point of the compass from the north, and with one foot of the compasses on B describe an arc intersecting BC in C: join AC, and the angle BAC will represent the ship's course. On the other hand, if a ship is to be sailed from the point A to a point on the line AC on a chart, draw meridian AB, describe arc BC with radius equal to chord of 60°, as before, and the extent from B to C, applied to the line of rhumbs, will give 2 pts. 3 qrs., denoting that the ship must be sailed by the compass N.N.E $\frac{3}{4}$ E.

The Line of Longitudes shows the number of equatorial miles in a degree of longitude on the parallels of latitude indicated by the degrees on the corresponding points of the line of chords. *Example.*—A ship in latitude 60° N. sailing E. 79 miles, required the difference of longitude between the beginning and end of her course. Opposite 60 on the line of chords stands 30 on the line of longitudes, which is, therefore, the number of equatorial miles in a degree of longitude at that latitude. Hence, as 30 : 79 :: 60 : 159 miles, the required difference of longitude.

The Lines of Sines, Secants, Tangents, and Semitangents are principally used for the several projections, or perspective representations, of the circles of the sphere, by means of which maps are constructed. Thus, the meridians and parallels of latitude being projected, the countries intended to be represented are traced out according to their respective situations and extent, the position of every point being determined by the intersection of its given meridian and parallel of latitude.

The plane upon which the circles are to be delineated is called the primitive, and the circumference of a circle, described with a radius representing upon the reduced scale of the drawing, the radius of the sphere, is called the circumference of the primitive. Lines, drawn from all the points of the circles to the eye, by their intersection with the primitive, form the projection.

When the eye is supposed to be infinitely distant, so that the lines of vision are parallel to one another and perpendicular to the primitive, the projection is called orthographic. When the primitive is a tangent plane to the sphere, and the eye is supposed to be at the center of the sphere, the projection is called gnomonic. When the eye is supposed to be at the surface of the sphere, and the primitive to pass through the center, so as to have the eye in its pole, the projection is called stereographic.

The projection is further termed equatorial, meridianal, or horizontal, according as the primitive coincides with, or is parallel to, the equator, or the meridian or horizon of any place.

To delineate the Orthographic Projection of the Circles of the Terrestrial Sphere upon the Plane of the Meridian of any place.—With a radius according to the contemplated scale of the projection, describe the circle W N E S, Fig. 23, for the circumference of the primitive, and draw the vertical and horizontal diameters N S and W E, which will be the projections of a meridian perpendicular to the primitive, and of the equator, respectively. Take out from the line of sines the sines of the latitudes through which the parallels are to be drawn, and reducing these sines to the radius of the primitive,* set off these reduced distances both ways from the center upon the line N S; and also both ways from the center upon the line W E, for the sines of the angles which the meridians, to be drawn at the same intervals as the parallels make with the meridian N S. Through the divisions thus set off, upon the line N S, draw straight lines parallel to W E, and such straight lines will be the projections of the several parallels of latitude, which are to be numbered 0 to 90, from the equator to either pole for the latitudes. With distances from the center to the divisions set off upon W E, as semi-minor axes, and the distance from C to N or S, equal to radius of primitive, as a common major axis, describe semi-ellipses,* and they will be the projections of the several meridians, which are to be numbered either way from the first meridian for the longitudes. In the figure the primitive coincides with the plane of the meridian of a place in 30° west longitude, or 150° east longitude, the sum of these two being 180°, as must always be the case.

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* If the proportional compasses be set in the proportion of the sine 90° on the line of sines to the radius of the primitive, one pair of points will give, reduced to this radius, the sines taken off by the other pair of points. The manner of taking from the sector a sine to any radius will be hereafter pointed out.

To delineate the Gnomonic Projection of the Circles of the Terrestrial Sphere upon a Plane parallel to the Equator, Fig. 24.—In this case the meridians will all be projected into straight lines, making the same angles one with another that their originals do on the surface of the sphere; the projection of the pole will be the center of the primitive, and the projections of the parallels of latitude will be circles described from the projection of the pole, as center, with distances equal to the tangents of the respective colatitudes reduced to the radius of the primitive. The parallel of 45° will, therefore, coincide with the circumference of the primitive, the parallels of latitudes greater than 45° will lie within the primitive, and for latitudes less than 45° the parallels will fall without the primitive, the radii of their projections increasing as the latitude decreases until the radius for projecting the equator becomes infinite. Describe, then, a circle for the primitive; draw straight lines radiating from its center, and equally inclined one to another for the projections of equidistant meridians; and number them 0 to 180 both ways from the first meridian for the longitudes. With the tangents of the colatitudes, taken at intervals equal to the angle between two successive meridians, and reduced to the radius of the primitive, as distances, describe from the center of the primitive concentric circles; and number them 90 to 45 from the pole to the primitive for the latitudes, containing the graduation beyond for the lower latitudes.

The gnomonic projection affords a good representation of the polar regions, but all places in latitudes lower than 60° appear greatly distorted. The gnomonic projection enlarges the representations of places at a distance from the center of projection beyond their proportionate true dimensions; and the orthographic, on the contrary, unduly contracts them; while both are adapted for representing best the countries at only a moderate distance from the center of projection.

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* These semi-ellipses may be thus described. From any point p, upon the straight edge of a piece of paper set off $p\,c$, equal the major axis, and $p\,b$, equal to the minor axis: then move the paper into various positions, but so that the point c, may always be upon the line W E, and the point b, upon the line N S, and the point p, will, in every such position, coincide with a point in the required ellipse.

To delineate the Stereographic Projection of the Circles of the Terrestrial Sphere upon the Horizon of any place, Fig. 26.—With radius determined upon, describe a circle for the primitive, and draw its vertical and horizontal diameters N S and W E, which will be the projections of the meridian of the place and of the prime vertical respectively. From the center *c*, set off upon the radius C S, produced, if necessary, the distance *a c*, equal to the tangent of the latitude of the place reduced to the radius of the primitive; and with center A, and distance *a* W, or *a* E, describe the circle W N E, which will be the projection of the meridian at right angles to N S, the meridian of the place; and consequently N, will be the projection of the pole. Through A, draw the right line A B, at right angles to A C, and another line A D, making any convenient angle with A B, and setting off A B, equal to radius of the primitive, and A D equal to the sine of the colatitude, taken from the line of sines, join B D. Now take from the line of tangents the angles which the other meridians to be drawn are to make with the meridian W N E, or the complements of the angles which they are to make with N S, and set them off both ways from A, upon the line A D; through each of the divisions L, thus found, draw L O, parallels to B D, and we have at O, the centers of the circles for describing the meridians. With centers O, and distances *o* N, describe the meridians, and number 0 to 180, both ways, from the first meridian, for the longitudes. For a parallel through any given latitude, take the difference of the complement of the given latitude and of the colatitude of the place from the line of semitangents, and having reduced it to the radius of the primitive, set off at *r* from C, towards N, for latitudes greater than the latitude of the place, and from C towards S, for latitudes less than the latitudes of the place:—again, take the sum of the complement of the given latitude and of the colatitude of the place from the line of semitangents, and set it off at *s*, from C, upon C N, produced: then the circle described upon *r s*, as diameter will be the parallel required. Draw these parallels for intervals of latitude equal to the angles made by two successive meridians, and number them 90 to 0 from the pole N, for the north latitudes, and again increasing from 0 on the other side of the equator for the south latitudes, if the place be in north latitude—or the converse, if the place be in south latitude.

The practical application of the preceding methods of projection is usually confined to the representation of an entire hemisphere, or at least of a considerable portion of a sphere; but for laying down smaller portions of the sphere the method of development may be advantageously adopted. In this method the portion of the sphere to be represented is considered as coincident with a portion of a cone, touching the sphere in a circle which is the middle parallel of latitude of the country to be represented, and this portion of the cone when developed forms a portion of a sector of a circle.

To lay down the meridians and parallels of latitude for this development. 1. Take a straight line, B C A, for the middle meridian of the intended map, and divide it into equal parts, to represent degrees and minutes of latitude according to the scale determined upon for the map. 2. From one of these divisions, A, which is conveniently situated to form the center of the map, set off from A to C the cotangent of the middle latitude, reduced to a radius equal to 57.3 of the divisions previously marked off as degrees, or to 3438 of those marked off as minutes. 3. With C as a center and radius C A, describe the arc D A E for the middle parallel of latitude, and divide it into equal parts to represent degrees and minutes of longitude, the lengths of these parts having, to the lengths of the parts previously set off on the meridian for degrees and minutes of latitude, the ratio cosine of middle latitude : radius. 4. With C as center, describe concentric arcs, through the divisions on C E, for the parallels of latitude; and draw straight lines, radiating from C, through the divisions on D A E for the meridians.

In our figure the middle latitude is 55°; A B is equal to the length of 57.3°, or the radius of the sphere; A C is equal to the cotangent of 55, or the tangent of 35 reduced to this radius; and C, consequently, is the center for describing the parallels, and the radiating point for the meridians.

In drawing a map of small extent, it is usual to make all the meridians and parallels of latitude straight lines; and to make the extreme parallels, and the meridian passing through the center of the map, proportional to their real magnitude.

Another and more exact method is, to make the meridian passing through the center of the map, and all the parallels of latitude, straight lines, as in the last method. Then all the degrees on each of the parallels are made proportional to their magnitude, and the lines passing through the corresponding points of division on the parallels will represent the meridians. These will be curved lines, and not straight, as in the last method. This is usually called *Flamstead's* Projection, as it was first used by that astronomer in constructing his "Celestial Atlas;" and it is extremely useful in geographical maps for countries lying on both sides of the equator.

A considerable improvement of this method, for countries of large extent, is to represent all the parallels of latitude by concentric circles, according to the principles of the conical development; and then to lay off the degrees on each parallel, proportional to their magnitude; that is, the degrees on each parallel must have to a degree of latitude the ratio of radius: cosine of the latitude of the parallel; and draw lines through the corresponding divisions of these parallels to represent the meridians. This delineation, perhaps, will give the different parts of a map of some extent in as nearly their due proportions as the nature of the case will admit.

We will now briefly explain the manner of constructing some of the simplest dials by means of the dialling scales.

To construct a Horizontal Dial, Fig. 28.—Draw parallel two lines, *a b*, *c d*, as a double meridian line, at a distance apart, equal to the thickness of the intended style, or gnomon (on your dial plate). Intersect it at right angles by another line, *e f*, called the six o'clock line. From the scale of latitudes take the latitude of the place with the compasses, and set that extent from *c* to *e* and from *a* to *f* on the six o'clock line, and then, taking the whole of six hours between the parts of the compasses from the scale, with this extent set one foot in the point *e*, and with the other intersect the meridian line *c d* at *d*. Do the same from *f* to *b*, and draw the right lines *e d* and *f b*, which are of the same lengths as the scale of hours. Place one foot of the compasses on the beginning of the scale, and extending the other to any hour on the scale, lay these extents off from *d* to *e* for the afternoon hours, and from *b* to *f* for the forenoon. In the same manner the quarters or minutes may be laid down if required. The edge of a ruler being now placed on the point *c*, draw the first five afternoon hours from that point through the marks on the line *d e*, and continue the lines of 4 and 5 through the center *c* to the other side of the dial for the like hours of the morning; lay a ruler on the point *a*, and draw the last five forenoon hours through the marks on the line *f b*, continuing the hour lines of 7 and 8 through the center *a* to the other side of the dial, for the evening hours, and figure the hours to the respective lines.

To make the Gnomon, Fig. 29.—From the line of chords, always placed on the same dialling scale, take the extent of 60°, and describe from the center *a* the arc *g n;* then with the extent of the latitude of the place, suppose London, 51½°, taken from the same line of chords, set one foot in *n*, and cross the arc with the other at *g*. From the center at *a* draw the line *a g* for the axis of the gnomon *a g i*, and from *g* let fall the perpendicular *g i* upon the horizontal meridian line *a n*, and there will be formed a triangle *a g i*. A plate or triangular frame similar to this triangle, and of the thickness of the interval of the parallel lines *a c* and *b d*, being now made and set upright between them, touching at *a* and *b*, its hypothenuse or axis *a g* will be parallel to the axis of the earth when the dial is fixed truly, and will cast its shadow on the hour of the day.

To make an erect South Dial, Fig. 30.—Take the complement of the latitude of the place, which for London is 90° less 51½ = 38½, from the scale of latitudes, and proceed in all other respects for the hour lines, as above, for the horizontal dial; only reversing the hours, and limiting them to the 7; and for the gnomon making the angle of the style's height equal to the colatitude 38½.

To construct an East or West Dial.—Draw the two meridian lines as before, and intersect it at right angles by another line, upon which set off, from the meridian lines, the tangents of 15°, 30°, 45°, &c., for every 15 degrees, reduced to a radius equal to the intended height of the style. The hour lines are to be drawn through the divisions thus marked, parallel to the meridian lines, and the meridian lines themselves are six o'clock hour lines. The gnomon is a plate in the form of a parallelogram, the breadth of which forms the height of the style of gnomon, and must be equal to the radius to which the tangents have been set off on the dial plate. It is set up between the meridian lines, perpendicular to the dial plate; and the dial is set up, so that the meridian lines, and consequently the edge of the gnomon, may *be parallel* to the earth's axis. As the sun only shines on the dial during half the day, if the dial fronts the east, it points out the time from sunrise to noon, or if the dial fronts the west, from noon to night.

GUNTER'S LINES.—These lines are graduated so as to form a scale of the logarithms of numbers, sines, and tangents; to which are sometimes added, for the use of the navigator, lines of the logarithms of the sine rhumbs and tangent rhumbs. They may be constructed as follows:—

1. *To construct the Line of Logarithmic Numbers marked N.*—Having fixed upon a convenient length for the entire scale, which must be exactly equal to the length of twenty of the primary divisions of the diagonal or vernier scale, or of the beam compasses, by which it is to be divided, bisect it, and figure it 1 at the commencement on the left hand, 1 again in the middle, and 10 at the end. The half line, then, is taken for unity, or the logarithm of 10, and, consequently, the whole line represents 2, or the logarithm of 100. The lengths corresponding to the three first figures of the logarithms of 2, 3, &c., up to 9, as found in the common table of logs., may now be taken off from the diagonal scale, or the length corresponding to four or even five figures may be estimated upon a vernier scale, or upon the beam compasses, if the scale be not less than twenty inches in length. These lengths are to be set off from the 1 at the commencement of the line for the logarithms of 2, 3, &c., to 9, and again from the 1 at the middle of the line for the logarithms of 20, 30, &c., to 90. The divisions thus formed are to be subdivided by setting off, in the same manner, the three, four, or five first figures of the logarithms of 1·1, 1·2, 1·3, &c., to 1·9; of 2·1, 2·2, 2·3, &c., to 2·9, and so on, each of the primary divisions being thus subdivided into ten; and these again are to be subdivided each into ten, or five, or two, as the length of the secondary divisions may admit, by setting off the logarithms of 1·11, 1·12, 1·13, &c.; or of 1·12, 1·14, &c.; or of 1·15, 1·25, &c.; and the scale is completed.

2. *To construct the Line of Logarithmic Sines marked S, Fig.* 32.—The whole length of the scale is taken as the logarithm of the radius, and, since this extent upon the line of numbers represents 2, or the logarithm of 100, it follows that the lines of sines, tangents, &c., are to be scales of the logarithms of the sines, tangents, &c., to radius 100, of which the logarithm is 2: whereas the logarithmic tables of sines, tangents, &c., are set down to a radius, of which the logarithm is 10. By taking 8, then, from each of the tabulated values of the logarithmic sines, tangents, &c., we should obtain the logarithmic sines, tangents, &c., to radius 100, and the three, four, or five first figures of these reduced values are to be set off, from the left hand towards the right, by one of the scales, or by the beam

compasses, as explained in the construction of the line of numbers; 1st, for every 10 degrees, then for every degree, and then for every half degree, every 10 minutes, and every 5 minutes, as far as the length of the several primary divisions will admit. The line is then numbered 1, 2, 3, &c., at every degree to 10, and afterwards 20, 30, 40, &c., at every ten degrees to 90, which stands at the extreme right, since sine 90° equals radius.

The tabulated logarithmic sine of 34′ 23″, being 8·0000669, will coincide, or nearly so, with the zero point upon our scale, and consequently angles smaller than this cannot be taken off from the sines. This remark applies equally to the line of tangents, the tabulated logarithmic tangent of 34′ 23″ being 8·0000886.

By taking the extents backwards from right to left, and reckoning them as forward distances, the line of sines becomes a line of cosecants, giving us, in fact, the excesses of the logarithmic cosecants above the logarithmic radius; and, by taking the complements of the required angles, the line of sines becomes a line of cosines when measured forwards from left to right, and a line of secants when measured backwards from right to left.

3. *To construct the Line of Logarithmic Tangents marked T.*—8 being taken from each of the tabulated values of the logarithmic tangents up to 45 degrees, the extents corresponding to these values are to be set off upon the scale, and numbered from left to right, in a similar manner to that in which the logarithmic sines were set off and numbered upon the line of logarithmic sines. The logarithmic tangent of 45° extends to the extreme right of the scale, coinciding in extent with the sine of 90, since tangent 45° equals radius, and the logarithmic tangents of the angles from 45° to 90 are measured backwards from the extreme right to the complement of the angle required, these extents giving us, in fact, the excesses of the logarithmic tangents sought above the logarithmic radius. When, then, the angle is greater than 45, the distance from radius to the angle, though measured backwards upon the scale, must be reckoned a forward distance, and *vice versa.*

The lines of logarithmic sine rhumbs, marked S. R., and tangent rhumbs, marked T. R., are formed in the same way as the lines of logarithmic sines and tangents, but are set off for the angles corresponding to the points and quarter points of the compass, instead of for degrees and minutes.

We shall now proceed to explain the uses of Gunter's lines.

1. *The Line of Logarithmic Numbers.*—The primary divisions upon this line, as explained in its construction, represent the logarithms of all the integers from 1 to 100, while the extents to the first subdivisions will indicate tenths of an unit from the beginning of the scale to 1 in the middle, and units from 1 in the middle to 10 at the end, where the figures 2, 3, &c., stand for 20, 30, &c., as has been explained in the construction. If any of the subdivisions be further subdivided into ten parts, each of these last divisions will indicate hundredths of an unit from 1 at the beginning to 1 in the middle, and tenths of an unit from 1 in the middle to 10 at the end. Upon pocket sectors, however, upon which Gunter's lines are now usually placed, affording a greater extent for the purpose than the six-inch plain scale, only the part from 1 in the middle to 2 towards the right is a second time divided, and that but into five parts instead of ten, every one of which must be accounted as two-tenths. By this line the multiplication and division of numbers of any denomination either whole or fractional may be readily accomplished, questions in proportion solved, and all operations approximatively performed with great rapidity, which can be performed by the common table of logarithms; but the numbers sought must always be supposed to be divided or multiplied by 10 as many times as will reduce them to the numbers, the logarithms of which are actually set off upon the line of numbers, and these tens must be mentally accounted for in the result.

Multiplication is performed by extending from 1 on the left to the multiplier; and this extent will reach forwards from the multiplicand to the product. Thus, if 125 were given to be multiplied by 250, extend the compasses from 1 at the left hand to midway between the second and third subdivision, in the first primary division from 1 to 2, for the 125. This extent is really the logarithm of 1·25. Set off this extent towards the right from the fifth subdivision after the primary division marked 2, which is taken to represent the log. of 250, but is really the log. of 2·5, and the compasses will reach to a quarter of the next subdivision beyond the first subdivision after the primary division marked 3. The extent of this point is really the logarithm of 3·125; but in this case it represents the number 31250, because the two powers of ten have been cast out from both the multiplier and multiplicand, and therefore the product must be multiplied by the product of four tens, or ten thousand; or, in other words, the first figure of the product must be reckoned as so many tens of thousands.

Division, being the reverse of multiplication, is performed by extending from 1 on the left to the divisor; and this extent will reach backwards from the dividend to the quotient. Thus, if 31250 were to be divided by 250, extend the compasses from 1 on the left to 2·5, and this extent will reach backwards from 3·125 to 1·25. Then, since the divisor contained 2 powers of ten and the dividend 4, the quotient must contain 2, and therefore the result is 125.

Proportion being performed by multiplication and division, extend the compasses from the first term to the second, and this extent will reach from the third to the fourth, taking care to measure in the same direction, so that, if the first be greater than the second, the third may be greater than the fourth, and *vice versa*. *Example.*—If the diameter of a circle be 7 inches, and the circumference 22, what is the circumference of another circle, the diameter of which is 10? Extend the compasses from 7 to 10, and this extent will reach from 22 to 31·4, or nearly 31½ inches, the circumference required.

The same thing may also be performed by extending from the first term to the third, and this extent will reach from the third term to the fourth. Thus, the extent from 7 to 22 will reach from 10 to 31·4, as before.

To measure a Superficies, extend from 1 to either the breadth or length, both being reduced in the same denomination, and this extent will reach forwards from the length or breadth to the superficial content. *Example.*—Required the superficial content of a plank 27 feet long by 15 inches broad. Extend from 1 to 1·25, for 15 inches equals 1·25 feet, and this extent will reach from 27 feet to 33·75 feet, the superficial content required.

Second Method.—Extend from 12 to the number of inches in the breadth, and this extent will reach in the same direction from the number of feet in the length to the number of square feet in the superficial content. Thus the extent forwards from 12 to 15 will reach forwards from 27 to 33·75, as before; while the extent backwards from 12 to 9 will reach backwards from 27 to 20·25 or 20¼, showing the superficial content of a plank 27 feet long by 9 inches broad to be 20·25 or 20¼ feet.

To measure a Solid Content.—The breadth, depth, and length being all reduced to the same denomination, extend from 1 to either the breadth or depth, and this extent will reach from the depth or breadth forwards to a fourth number, which will represent the superficial content of the section at the place measured: then, if the breadth and depth be the same throughout the entire length, the extent from 1 to the superficial content thus found will reach forwards from the length to the solid content. *Example.*—What is the solid content of a pillar 1 foot 3 inches square, and 21 feet 9 inches long? The extent from 1 to 1·25 reaches forward from 1·25 to 1·56, the superficial content of a section of the pillar; and the extent from 1 to 1·56 reaches from 21·75 to 34, or more accurately to 33·93, the solid content in feet.

2. *The Lines of Logarithmic Sines and Tangents.*—These lines are generally used, in connection with the line of numbers, for solving all proportions in which any of the terms are functions of angles, as sines, tangents, &c., and, in fact, all questions in which such quantities appear as factors or divisors. We will exemplify their use by giving the solution, by their aid, of the several cases of right-angled trigonometry.

Case 1. The hypothenuse and angles being given, to find the perpendicular and base.

Note.—One acute angle of a right-angled triangle being the complement of the other, or the sum of the two acute angles being equal to 90°, when one of the acute angles is given, the other is also given.

Solution.—Extend the compass from 90°, or radius, on the line of sines to the number of degrees in either of the acute angles, and that extent will reach backwards, on the line of numbers, from the hypothenuse to the side opposite this angle. *Example*, *Fig.* 33.—Given the hypothenuse AB = 250, and the angle A = 35° 30′.

$$\begin{array}{r} 90^\circ\ \ 0' \\ \mathrm{BAC} = \underline{35\ \ 30'} \\ \mathrm{ABC} = 54\ \ 30' \end{array}$$

Extend from 90° to 35° 30′ on the line of sines, and this extent will reach from 250 to 145 on the line of numbers.	∴ BC = 145
Extend from 90° to 54° 30′ on the line of sines, and this extent will reach from 250 to 203·5 on the line of numbers.	∴ AC = 203·5

Case 2. The angles, and one side being given, to find the hypothenuse, and the other side.

Solution.—Extend from the angle opposite the given side to 90°, or radius, on the line of sines, and this extent will reach forwards from the given side to the hypothenuse on the line of numbers. Again, extend from the angle opposite the given side to the angle opposite the required side, and this extent will reach in the same direction on the line of numbers, from the given side to the required side. Or, extend from radius, or 45°, on the line of tangents, to the angle opposite the required side, and the extent will reach, in the same direction on the line of numbers, from the given side to the required side; recollecting that, when the angle is greater than 45°, the extent is to be taken on the scale backwards from rad. or 45° to the complement of the angle, but is to be reckoned a forward distance, the logarithmic tangents of angles of greater than 45° exceeding the logarithmic tangent of 45°, or radius, by as much as the logarithmic tangents of their complements fall short of it. *Example.*—Given the angle A = 35° 30′ and side AC – 203·5.

90° 0′
Extend from 54° 30′ to 90°, or rad., upon the line of BAC = 35 30
sines, and this extent will reach forwards from 203·5 ABC = 54 30
to 250 on the line of numbers. . . . . . . ∴ AB = 250

Again, extend from 54° 30′ backwards to 35° 30′ on the
line of sines, and this extent will reach backwards
from 203·5 to 145 on the line of numbers. . . ∴ BC = 145

Or extend backwards from 45°, rad., to 35° 30′ on the line of tangents, and this extent will reach backwards from 203·5 to 145 on the line of numbers, as before.

Case 3. The hypothenuse and one side being given, to find the angles and the other side.

Solution.—Extend from the hypothenuse to the given side on the line of numbers, and this extent will reach from 90 or rad. to the angle opposite the given side upon the line of sines. The other angle is the complement of this. Extend upon the line of sines from the rad. to the angle last found, which is opposite the required side, and this extent will reach from the hypothenuse to the required side. *Example.*—Given the hypothenuse AB = 250, and the side AC = 203·5.

Extend backward from 250 to 203·5 on the line of
numbers, and this extent will reach from 90° to 54° 30′ 90° 0′
on the line of sines. . . . . ∴ ABC = 54 30
BAC = 35 30

Extend from 90 to 35° 30′ on the line of sines, and this extent will reach backwards from 250 to 145 on the line of numbers. . . . . ∴ B C = 145

Case 4. The two sides being given, to find the angles and the hypothenuse.

Solution.—Extend from one side to the other upon the line of numbers, and this extent will reach backwards upon the line of tangents from rad. to the least angle, and to the same point, considered as a forward distance, representing the greatest angle, which is the complement of the least. Again, extend on the line of sines from one of the angles just found to rad., and this extent will reach from the side opposite the angle taken to the hypothenuse. *Example.*—Given A C = 203·5 and B C = 145.

Extend backwards upon the line of numbers from 203·5 to 145, and this extent will reach backwards from 45° to 35° 30′ on the line of tangents, which is the angle opposite the side 145. . . . . . . 90° 0′ ∴ B A C = 35 30

If we measure forwards from 145 to 203·5, then from rad. to 35° 30′ is to be considered a forward distance, and the angle to be taken as the complement of 35° 30′, that is, 54° 30′, which is the angle opposite the side 203·5. . . . . ∴ A B C = 54 30

Again, extend from 30° 23′ to 90° on the line of sines, and this extent will reach from 145 to 250 upon the line of numbers. . . . . ∴ A B = 250.

THE PLANE SCALE, FIG. 35, being such a one as is usually supplied with a pocket case of instruments. It is made of ivory, six inches long, and one inch and three-quarters broad. On the face of the instrument represented in the engraving, a protractor is formed round three of its edges for readily setting off angles. In using this protractor, the fourth edge, which is quite plane, with the exception of a single stroke in the middle, is to be made to coincide with the line from which the angle is to be set off; and the stroke in the middle with the point in this line, at which the angle is to be set off; a mark is then to be made with the pricking point, at the point of the paper which coincides with the stroke on the protractor, marked with the number of degrees in the angle required to be drawn; and, the protractor being now removed, a straight line is to be drawn through the given point in the given line, and the point thus pricked off. The instrument has on the same face the two diagonal scales already described, and on the opposite face scales of equal parts, and several of the protracting scales already described, according to the purposes to which the scale is to be applied: thus, for laying down an ordinary survey, we merely require scales of equal parts, and a line of chords, and these consequently, are all the lines placed on many of the instruments in the pocket cases; but for projecting maps, lines of sines, tangents and semitangents are required, for dialling, the dialling lines, and for the purposes of the navigator the lines of rhumbs, and longitudes, the whole of Gunter's lines already described, and two lines of meridianal, and equal parts to be used together in laying down distances, &c., upon Mercator's charts. The plane scale is sometimes fitted with rollers, as represented in our engraving, making it at the same time a convenient small parallel rule.

THE SECTOR, FIG. 36.—This valuable instrument may well be called an universal scale. By its aid all questions in proportion may be solved; lines may be divided either equally or unequally into any number of parts that may be desired; the angular functions, viz., chords, sines, tangents, &c., may be set off or measured to any radius whatever; plans and drawings may be reduced or enlarged in any required proportion; and, in short, every operation in geometrical drawing may be performed by the aid of this instrument, and the compasses only

The name sector, is derived from the tenth definition of the third Book of Euclid, in which this name is given to the figure contained by two radii of a circle, and the circumference between them. The instrument consists of two equal rulers, called legs, which represent the two radii, movable about the center of a joint, which center represents the center of the circle. The legs can consequently be opened so as to contain any angle whatever, or completely opened out until their edges come into the same straight line.

Sectors are made of different sizes, and their length is usually denominated from that of the legs when shut together. Thus, a sector of six inches, such as is supplied in the common pocket cases of instruments, forms a rule of 12 inches, when opened; and this circumstance is taken advantage of, by filling up the spaces not occupied by the sectoral lines with such lines as it is most important to lay down upon a greater length than the six-inch plane scale will admit. Among these the most usual are (1) the lines of logarithmic numbers, sines, and tangents already described (2,) a scale of 12 inches, in which each inch is divided into ten equal parts; and (3) a foot divided into ten equal primary divisions, each of which is subdivided into ten equal parts, so that the whole is divided into 100 equal parts. The last-mentioned is called the decimal scale, and is placed on the edge of the instrument.

The *sectoral lines* proceed in pairs from the center, one line of each pair on either leg, and are, upon one face of the instrument, a pair of scales of equal parts, called the *line of lines*, and marked L; a pair of lines of chords, marked C; a pair of lines of secants, marked S; a pair of lines of polygons, marked POL. Upon the other face, the sectoral lines are—a pair of lines of sines, marked S; a pair of lines of tangents up to 45°, marked T; and a second line of tangents to a lesser radius, extending from 45° to 75°.

Each pair of sectoral lines, except the line of polygons, should be so adjusted as to make equal angles at the center, so that the distances from the center to the corresponding divisions of any pair of lines, and the transverse distance between these divisions, may always form similar triangles. On many instruments, however, the pairs of lines of secants, and of tangents from 45° to 75°, make angles at the center equal to one another, but unequal to the angle made by all the other pairs of lines.

The solution of questions on the sector is said to be *simple*, when the work is begun and ended upon the same pair of lines; *compound*, when the operation is begun upon one pair of lines and finished upon another.

In a compound solution the two pairs of lines used must make equal angles at the center, and consequently, in the exceptional case mentioned above, the lines of secants and of tangents above 45° cannot be used in connection with the other sectoral lines.

When a measure is taken on any of the sectoral lines beginning at the center, it is called *a lateral* distance; but, when a measure is taken from any point on one line to its corresponding point on the line of the same denomination on the other leg, it is called a *transverse* or *parallel* distance.

The divisions of each sectoral line are contained within three parallel lines, *the innermost* being the line on which the points of the compasses are to be placed, because this is the only line of the three which goes to the center, and is therefore the sectoral line.

On the Principle of the Use of the Sectoral Lines.—1*st. In the case of a Simple Solution, Fig.* 37.—Let the lines A B, A E, represent a pair of sectoral lines, and B C, D E, any two transverse distance taken on this pair of lines; then, from the construction of the instrument, we have A B equal to A C, and A D equal to A E, so that A B : A C : : A D : A E, and the triangles A B C and A D E, have the angle at A, common, and the sides about this common angle proportional; they are, therefore, similar, and—

A B : B C : : A D : D E.

In the case of a compound solution, the angles at A, are equal, but not common, and the reasoning is, in all other respects, exactly the same.

USES OF THE LINE OF LINES.—*To find a Fourth Proportional to three given Lines.*—Set off from the center a lateral distance equal to the first term, and open the sector till the transverse distances at the division thus found, expressing the first term, is equal to the second term; again, extend to a point whose lateral distance from the center is equal to the third term, and the transverse distance at this point will be the fourth term required.

If the legs of the sector will not open far enough to make the lateral distance of the second term a transverse distance at the division expressing the first term, take any aliquot part of the second term, which can conveniently be made such transverse distance, and the transverse distance at the third term will be the same aliquot part of the fourth proportional required.

A third proportional to two given lines is found by taking a third line equal to the second, and finding the fourth proportional to the three lines.

Example.—To find a fourth proportional to the numbers 2, 5, and 10. Open the sector till the lateral distance of the second term 5, becomes the transverse distance at 2, the first term; then the transverse distance at 10 will extend, as a lateral distance, from the center to 25, the fourth proportional required.

To bisect a given Straight Line.—Take the extent of the line in the compasses, and open the sector till this extent is a transverse between 10 and 10 on the line of lines: then the transverse distance from 5 to 5, on the same pair of sectoral lines, gives the half of the line, and this extent set off from either end will bisect it.

To divide a Straight Line into any Number of equal Parts, Fig. 38.—When the number of parts are a power of 2, the operations are best performed by continual bisection. Thus, let it be required to divide the line A B into sixteen equal parts. 1. Make A B a transverse distance between 10 and 10 on the line of lines; then take off the transverse distance of 5 and 5, and set it off from A or B to 8, and A B will be divided into two equal parts at the division 8. 2. Make A 8 a transverse distance at 10, and then the transverse distance at 5, set off from A or 8 at 4, and from B or 8 at 12, will divide the line into four equal parts at the divisions 4, 8, and 12. 3. Make the extent A 4 a transverse distance at 10, and the transverse distance at 5 will again bisect each of the parts last set off, and divide the whole line into eight equal parts at the divisions 2, 4, 6, 8, 10, 12, and 14. Each of these may be again bisected by taking the transverse distance at 2½ or 2·5, that is, at the middle division between the 2 and the 3 upon the line of lines, and the line will be divided as required.

When the divisions become smaller than can be conveniently bisected by the method just explained, the operation may still be continued to any required extent by taking the extent of an odd number of the divisions already obtained as the transverse distance of 10 and 10, and setting off the half of this extent, or the transverse distance at 5, from the several divisions already obtained. Thus, in the preceding example, by making the extent of three of the divisions, or five, or seven, a transverse distance at 10, the transverse distance at 5, set off from the several divisions already obtained, will divide A B into 32 equal parts. 2. When the number of parts is not a power of 2, the operations cannot all be performed by bisections; but still, by a judicious selection of the parts into which the line is first divided, many of the after operations may be performed by bisections. *Example.*—Let it be required to divide the line A B, Fig. 39, into seven equal parts. 1. Make the whole extent, A B, a transverse distance between 7 and 7 on the line of lines; then take off the transverse distance of 4 and 4, and set it off from A and B to 4 and 3. 2. Make this extent from A to 4 a transverse distance at 10; then the transverse distance at 5 bisects A 4 and 3 B in 2 and 5; set off from 3, gives 1, and from 4 gives 6; and thus the line A B is divided into seven equal parts as required.

To open the Sector so that the Line of Lines may answer for any required Scale of equal Parts.—Take one inch in the compasses, and open the sector, till this extent becomes a transverse distance at the division indicating the number of parts in an inch of the required scale; or, if there be not an integral number of parts in one inch, it will be better to take such a number of inches as will contain an integral number of parts, and make the extent of this number of inches, if it be not too great, a transverse distance at the division indicating the number of parts of the required scale in this extent.

Example.—To adjust the Sector as a Scale of One Inch to Four Chains.—Make one inch the transverse distance of 4 and 4; then the transverse distances of the other corresponding divisions and subdivisions will represent the number of chains and links indicated by these divisions: thus, the transverse distance from 3 to 3 will represent three chains; the transverse distance at 4·7, or the seventh principal subdivision after the primary division marked 4, will represent 4 chains 70 links, and so on.

To construct a Scale of Feet and Inches in such a manner that an extent of Three Inches shall represent Twenty Inches.—1. Make three inches a transverse distance between 10 and 10, and the transverse distance of 8 and 8 will represent 16 inches. 2. Set off this extent from A to B, Fig. 40, divide it by continual bisection into 16 equal parts, and place permanent strokes to mark the first 12 of these divisions, which will represent inches. 3. Place the figure 1 at the twelfth stroke, and set off again the extent of the whole 12 parts, from 1 to 2, 2 to 3, &c., to represent the feet.

As an Example of the use of the Line of Lines in reducing Lines, let it be required to reduce a Drawing in the Proportion of 5 *to* 8.—Take in the compasses the distance between two points of the drawing, and make it a transverse distance at 8 and 8; then the transverse distance of 5 and 5 will be the distance between the two corresponding points of the copy. 2. These two points having been laid down, make the distance between one of them and a third point a transverse distance at 8, and with the transverse distance at 5 describe, from that point as center, a small arc. 3. Repeat the operation with the other point, and the intersection of the two small arcs will give the required position of the third point in the copy. In the same manner all the other points of the reduced copy may be set off, each one from two points previously laid down.

Line of Chords.—The double scales of chords upon the sector are more generally useful than the single line of chords described on the plane scale; for, on the sector, the radius with which the arc is to be described may be of any length between the transverse distance of 60 and 60 when the legs are close, and that of the transverse of 60 and 60 when the legs are opened as far as the instrument will admit of: but, with the chords on the plane scale, the arc described must be always of the same radius.

To protract or lay down a right-lined Angle B A C, *which shall contain a given number of Degrees, suppose* 46°.—*Case* 1. When the angle contains less than 60°, make the transverse distance of 60 and 60 equal to the length of the radius of the circle, and with that opening describe the arc B C. Fig. 41. Take the transverse distance of the given degrees 46, and lay this distance on the arc from the point B to C. From the center A of the arc draw two lines A C, A B, each passing through one extremity of the distance B C laid on the arc; and these two lines will contain the angle required. *Case* 2. When the angle contains more than 60°. Suppose, for example, we wish to form an angle containing 148°. Describe the arc B C D, and make the transverse distance of 60 and 60 equal the radius as before. Take the transverse distance of ½ or ⅓, &c., of the given number of degrees, and lay this distance on the arc twice or thrice, as from B to *a*, *a* to *b*, and from *b* to D. Draw two lines connecting B to A, and A to D, and they will form the angle required.

When the required angle contains less than 5°, suppose 3½, it will be better to proceed thus. With the given radius, and from the center A, describe the arc D, G; and from some point, D, lay off the chord of 60°, which suppose to give the point G, and also from the same point D lay off in the same direction the chord of 56½° (= 60° − 3½°), which would give the point E. Then through these two points E and G, draw lines to the point A, and they will represent the angle of 3½° as required.

From what has been said about the protracting of an angle to contain a given number of degrees, it will easily be seen how to find the degrees (or measure) of an angle already laid down.

Line of Polygons.—The line of polygons is chiefly useful for the ready division of the circumference of a circle into any number of equal parts from 4 to 12; that is, as a ready means to inscribe regular polygons of any given number of sides, from 4 to 12, within a given circle. To do which, set off the radius of the given circle (which is always equal to the side of an inscribed hexagon) as the transverse distance of 6 and 6, upon the line of polygons. Then the transverse distance of 4 and 4 will be the side of a square; the transverse distance between 5 and 5, the side of a pentagon; between 7 and 7, the side of a heptagon; between 8 and 8, the side of an octagon; between 9 and 9, the side of a nonagon, &c., all of which is too plain to require an example.

If it be required to form a polygon, upon a given right line, set off the extent of the given line, as a transverse distance between the points upon the line of polygons, answering to the number of sides of which the polygon is to consist; as for a pentagon between 5 and 5; or for an octagon between 8 and 8; then the transverse distance between 6 and 6 will be the radius of a circle whose circumference would be divided by the given line into the number of sides required.

The line of polygons may likewise be used in describing, upon a given line, an isosceles triangle, whose angles at the base are each double that at the vertex. For, taking the given line between the compasses, open the sector till that extent becomes the transverse distance of 10 and 10, then the transverse distance of 6 and 6 will be the length of each of the two equal sides of the isosceles triangle.

All such regular polygons, whose number of sides will exactly divide 360 (the number of degrees into which all circles are supposed to be divided) without a remainder, may likewise be set off upon the circumference of a circle by the line of chords. Thus, take the radius of the circle between the compasses, and open the sector till that extent becomes the transverse distance between 60 and 60 upon the line of chords; then, having divided 360 by the required number of sides, the transverse distance between the numbers of the quotient will be the side of the polygon required. Thus for an octagon, take the distance between 45 and 45; and for a polygon of 36 sides take the distance between 10 and 10, &c.

Lines of Sines, Tangents, and Secants.—*Given the Radius of a Circle (suppose equal to Two Inches;) required the Sine and Tangent of* 28° 30′ *to that Radius.*—Open the sector, so that the transverse distance of 90 and 90 on the sines, or 45 and 45 on the tangents, may be equal to the given radius, viz., two inches;

then will the transverse distance of 28° 30′, taken from the sines, be the length of that sine to the given radius, or, if taken from the tangents, will be the length of that tangent to the given radius.

But, if the Secant of 28° 30′ *is required*, make the given radius of two inches a transverse distance 0 and 0, at the beginning of the line of secants, and then take the transverse distance of the degrees wanted, viz., 28° 30.

A Tangent greater than 45° (*suppose* 60°) *is thus found.*—Make the given radius, suppose two inches, a transverse distance to 45 and 45, at the beginning of the line of upper tangents, and then the required degrees (60) may be taken from the scale.

The tangent, to a given radius, of any number of degrees greater than 45° can also be taken from the line of lower tangents, if the radius can be made a transverse distance to the complement of those degrees on this line.

Example.—To find the tangent of 78° to a radius of two inches. Make two inches a transverse distance at 12 on the lower tangents, then the transverse distance of 45 will be the tangent of 78°.

In like manner the secant of any number of degrees may be taken from the sines, if the radius of the circle can be made a transverse distance to the complement of those degrees upon this line. Thus making two inches a transverse distance to the sine of 12°, the transverse distance of 90 and 90 will be the secant of 78°.

To find, by means of the lower tangents and sines, the degrees answering to a given line, greater than the radius which expresses the length of a tangent or secant to a given radius. For a tangent, make the given line a transverse distance at 45 on the lower tangents; then take the extent of the given radius, and apply it to the lower tangents; and the complement of the degrees at which it becomes a transverse distance will be the number of degrees required. For a secant make the given line a transverse distance at 90 on the sines; then the extent of the radius will be a transverse distance at the complement of the number of degrees required.

Given the Length of the Sine, Tangent, or Secant of any Degrees, to find the Length of the Radius to that Sine, Tangent, or Secant.—Make the given length a transverse distance to its given degrees on its respective scale. Then,

If a sine	the transverse distance of	90 and 90 on the sines	will be the radius sought.
If a tangent under 45°		45 and 45 on the tangents	
If a tangent above 45°		45 and 45 on the upper tangents	
If a secant		0 and 0 on the secants	

To find the Length of a versed Sine, to a given number of Degrees, and a given Radius.—1. Make the transverse distance of 90 and 90 on the sines equal to the given radius. 2. Take the transverse distance of the sine of the complement of the given number of degrees. 3. If the given number of degrees be less than 90, subtract the distance just taken, viz., the sine of the complement, from the radius, and the remainder will be the versed sine: but, if the given number of degrees are more than 90, add the complement of the sine to the radius, and the sum will be the versed sine.

To open the legs of a Sector, so that the corresponding double Scales of Lines, Chords, Sines, and Tangents may make each a right Angle.—*On the line of lines* make the lateral distance 10, a transverse distance between 8 on one leg, and 6 on the other leg.

On the line of sines make the lateral distance 90, a transverse distance from 45 to 45; or from 40 to 50; or from 30 to 60; or from the sine of any degree to their complement.

On the line of sines make the lateral distance of 45 a transverse distance between 30 and 30.

MARQUOIS'S SCALES, FIG. 3.—These scales consist of a right-angled triangle, of which the hypothenuse, or longest side, is three times the length of the shortest, and two rectangular rules. Fig. 51, which is drawn one-third the actual size of the instruments from which it is taken, represents the triangle and one of the rules, as being used to draw a series of parallel lines. Either rule is one foot long, and has, parallel to each of its edges, two scales, one placed close to the edge and the other immediately within this, the outer being termed the artificial, and the inner, the natural scale. The divisions upon the outer scale are three times the length of those upon the inner scale, so as to bear the same proportion to each other that the longest side of the triangle bears to the shortest. Each inner, or natural scale, is, in fact, a simply divided scale of equal parts having the primary divisions numbered from the left hand to the right throughout the whole extent of the rule. The first primary division on the left hand is subdivided into ten equal parts, and the number of these subdivisions in an inch is marked underneath the scale, and gives it its name. On one of the pair of Marquois's scales now before us, we have, on one face, scales of 30 and 60, on the obverse scales of 25 and 50, and on the other we have on one face scales of 35 and 45, and on the obverse scales of 20 and 40. In the artificial scales the zero point is placed in the middle of the edge of the rule, and the primary divisions are numbered both ways from this point to the two ends of the rule, and are, every one, subdivided into ten equal parts, each of which is, consequently, three times the length of a subdivision of the corresponding natural scale.

The triangle has a short line drawn perpendicular to the hypothenuse near the middle of it, to serve as an index or pointer; and the longest of the other two sides has a sloped edge.

To draw a Line parallel to a given Line, at a given Distance from it.—1. Having applied the given distance to the one of the natural scales which is found to measure it most conveniently, place the triangle with its sloped edge coincident with the given time, or rather at such small distance from it, that the pen or pencil passes directly over it when drawn along this edge. 2. Set the rule closely against the hypothenuse, making the zero point of the corresponding artificial scale coincide with the index upon the triangle. 3. Move the triangle along the rule, to the left or right, according as the required line is to be above or below the given line, until the index coincides with the division or subdivision corresponding to the number of divisions or subdivisions of the natural scale, which measures the given distance; and the line drawn along the sloped edge in its new position will be the line required. Fig. 42.

Note.—The natural scale may be used advantageously in setting off the distances in a drawing, and the corresponding artificial scale in drawing parallels at required distances.

To draw a Line perpendicular to a given Line from a given Point in it.—1. Make the shortest side of the triangle coincide with the given line, and apply the rule closely against the hypothenuse. 2. Slide the triangle along the rule until a line drawn along the sloped edge passes through the given point; and the line so drawn will be the line required.

The advantages of Marquois's scales are: 1st, that the sight is greatly assisted by the divisions on the artificial scale being so much larger than those of the natural scale to which the drawing is constructed: 2d, that any error in the setting of the index produces an error of but one-third the amount in the drawing.

If the triangle be accurately constructed, these scales may be advantageously used for dividing lines with accuracy and dispatch; figure 51, as well as the sliding rule (fig. 52), was drawn by the aid of Marquois's Scales alone.

THE VERNIER. (Figs. 49 and 50.)—The property of this ingenious little subsidiary instrument will be readily comprehended from what has been already said of the construction and use of a vernier scale. It is so constructed as to slide evenly along the graduated limb of an instrument, and enables us to measure distances, or read off observations with remarkable nicety. In the vernier scale before described, the divisions on the lower, or subsidiary scale, were longer than those on the upper or primary scale; but in the vernier now to be described, the divisions are usually shorter than those upon the limb to which it is attached, the length of the graduated scale of the vernier being exactly equal to the length of a certain number $(n-1)$ of the divisions upon the limb, and the number (n) of division upon the vernier being one more than the number upon the same length of the limb.

Let, then, L represent the length of a division upon the limb, and V, vernier:

so that $(n-1)\ \mathrm{L} = n\ \mathrm{V}$;

$$\text{and therefore } \mathrm{L} - \mathrm{V} = \mathrm{L} - \frac{n-1}{n}\mathrm{L} = \frac{1}{n}\mathrm{L};$$

or the defect of a division upon the vernier from a division upon the limb is equal to the nth part of a division upon the limb, n being the number of divisions upon the vernier.

In fig. 1, six divisions of the vernier are equal to five divisions of the limb, and, consequently, the above defect, or L – V, is equal to a sixth part of a division upon the limb, or to 20′, since a division of the limb is equal to 2°.

In fig. 2, ten divisions of the vernier are equal to nine divisions of the limb, and, consequently, L – V is equal to a tenth part of a division upon the limb, or to the hundredth part of an inch, a division of the limb being equal to the tenth part of an inch.

In reading off we must first look to the lozenge, as pointing out the exact place upon the limb at which the required measurement is indicated. If, then, the stroke upon the vernier at the lozenge exactly coincides with a stroke upon the limb, the reading at this stroke gives the measurement required; but, if the stroke at the lozenge be a distance beyond a stroke upon the limb, then will this distance be equal to once, or twice, or thrice, &c., the difference of a division upon the limb and upon the vernier, according as the stroke at the end of the first, or second, or third, &c., division upon the vernier coincides with a stroke upon the limb.

In fig. 1 the stroke upon the vernier at the lozenge falls beyond the stroke indicating 22° upon the limb, and the stroke at the end of the second division upon the vernier coincides with a stroke upon the limb; the reading therefore is 22° 40′.

In fig. 2, the stroke upon the vernier at the lozenge falls beyond the stroke indicating one inch and three-tenths upon the limb, and the stroke at the end of the sixth division upon the vernier coincides with a stroke upon the limb: the reading, therefore, is 1·36 inches, or one inch three-tenths and six hundredths.

The limbs of the best sextants are now divided at every 10 minutes, and 59 of these parts are made equal to 60 divisions of their verniers. In this case

$$\text{L} - \text{V} = \frac{\text{L}}{60} = \frac{10'}{60'} = 10''$$

so that these instruments can be read off by the aid of their verniers to an accuracy of 10 seconds. The verniers occupy on the limbs spaces equal to 9° 50′.

The limbs of small theodolites are generally divided at every 30 minutes, and 29 of these parts are made equal to 30 divisions of their verniers, which therefore enable us to read off to an accuracy of $\frac{30'}{30'}$, or 1′.

In the mountain barometer, the scale being divided into $\frac{5}{100}$ths of an inch, 9 of these parts are made equal to 10 divisions of the vernier, which therefore enables us to read off to an accuracy of $\frac{5}{1000}$ths of an inch.

In the above explanations we have only considered the case of an exact coincidence between some one of the strokes upon the vernier and a stroke upon the limb. Suppose now that in fig. 1 the stroke at the end of the second division, instead of coinciding with a stroke upon the limb, fell a little beyond it, while the stroke at the end of the third division fell a little short of a stroke upon the limb; then the measurement indicated would be something between 22° 40′ and 23°, which the observer, should there be no other mechanism attached to the vernier, must estimate by guess, according to the best of his judgment. By the aid, however, of a piece of mechanism, which is called *a micrometer*, and which we proceed to describe, the excess of the angle indicated above 22° 40′ might be exactly computed.

The instrument having been nearly set by the hand alone, the vernier is fixed in this position by turning a screw, called the clamping screw, which is shown on the top of the vernier in Fig. 2; but is not seen in Fig. 1, being at the back of the instrument. The instrument is then set more accurately by the screw at the side of the vernier, shown in both figures, which gives a slow motion to the vernier plate, and to the limb or index bar attached to the vernier. This screw is called a tangent or slow motion screw, and the *micrometer* consists of a graduated cylindrical head B B, attached to this screw, and an index I, attached to the vernier. Suppose, now, the tangent screw to be of that fineness, that, whilst it is turned once round, by means of the milled head H, so that the graduated head B B, makes one complete revolution, the vernier is advanced on the limb of the instrument, a distance equal to the difference of a division of the limb, and of the vernier; then, in Fig. 1, one revolution of the screw advances the vernier a distance equal to 20′; and, if the cylindrical head B B, be divided into 60 equal parts, a revolution of the screw through one of these parts would advance the vernier a distance equal to 20″.

Suppose, then, that in the illustration above given, the screw has to be turned back, so that 14 of these graduations pass the index I, in order to bring the stroke at the end of the second division upon the vernier into coincidence with a stroke upon the limb; then the corresponding space moved through by the vernier would be equal to 20″ × 14, or 4′ 40″, and the reading of the instrument would be 22° 44′ 40″.

Similarly, by means of a micrometer divided into ten equal parts, a distance to the thousandth part of an inch may be read off by the vernier in Fig. 2. If the micrometer in this case were divided into one hundred equal parts, a distance might be read off to the ten thousandth part of an inch; or the same effect may be produced by dividing the micrometer into ten equal parts, and making the screw of such firmness that ten complete revolutions move the vernier through a distance equal to the difference of a division of the limb and of the vernier, or the one hundredth part of an inch.

Beam Compasses.—Fig. 44 represents this instrument, which consists of a beam A A, of any length required, generally made of well-seasoned mahogany. Upon its face is inlaid throughout its whole length a slip of holly or boxwood *a a*, upon which are engraved the divisions or scale, either feet and decimals, or inches and decimals, or whatever particular scale may be required. Those made for the use of the persons engaged on the ordinance survey of Ireland were divided to a scale of chains, 80 of which occupied a length equal to six inches, which, therefore, represented one mile, six inches to the mile being the scale to which that important survey is plotted. Two brass boxes B and C, are adapted to the beam; of which the latter may be moved, by sliding to any part of its length, and fixed in position by tightening the clamp screw E. Connected with the brass boxes are the two points of the instrument G and H, which may be made to have any extent of opening by sliding the box C, along the beam, the other box B, being firmly fixed at one extremity. The object to be attained, in the use of this instrument, is the nice adjustment of the points G H, to any definite distance apart. This is accomplished by two vernier, or reading plates *b c*, each fixed at the side of an opening in the brass boxes to which they are attached, and affording the means of minutely subdividing the principal divisions *a a*, on the beam which appear through those openings. D is a clamp screw for a similar purpose to the screw E, namely, to fix the box B, and prevent motion in the point it carries after adjustment to position. F is a slow motion screw, by which the point G, may be moved any very minute quantity for perfecting the setting of the instrument, after it has been otherwise set as nearly as possible by the hand alone.

The method of setting the instrument for use may be understood from the above description of its parts, and also by the following explanation of the method of examining and correcting the adjustment of the vernier *b*, which, like all other mechanical adjustments, will occasionally get deranged. This verification must be performed by means of a detached scale. Thus, suppose, for example, that our beam compass is divided to feet, inches, and tenths, and subdivided by the vernier to hundredths, &c. First set the zero division of the vernier to the zero of the principal divisions on the beam, by means of the slow motion screw F. This must be done very nicely. Then slide the box C, with its point G, till the zero on the vernier *c*, exactly coincides with any principal division on the beam, as twelve inches or six inches, &c. To enable us to do this with extreme accuracy, some superior kinds of beam compasses have the box C, also furnished with a tangent or slow motion screw, by which the setting of the points or divisions may be performed with the utmost precision. Lastly, apply the points to a similar detached scale, and, if the adjustment be perfect, the interval of the points G H, will measure on it the distance to which they were set on the beam. If they do not, by ever so small a quantity, the adjustment should be corrected by turning the screw F, till the points do exactly measure that quantity on the detached scale; then, by loosening the little screws which hold the vernier *b*, in its place, the position of the vernier may be gradually changed, till its zero coincides with the zero on the beam; and, then tightening the screws again, the adjustment will be complete.

PLOTTING SCALES.—Plotting scales, also called feather-edged scales, are straight rulers, usually about ten or twelve inches long. Each ruler has scales of equal parts, decimally divided, placed upon its edges, which are made sloping, so that the extremities of the strokes marking the divisions lie close to the paper. The primary divisions represent chains, and the subdivisions, consequently, ten links each, as there are 100 links on the surveying chain. Plotting scales may be procured in sets, each with a different number of chains to the inch.

The advantages of this arrangement are, that the distances required can be transferred with great expedition from the scale to the paper by the aid of the pricking point alone, and the marks denoting the divisions are in no danger of becoming defaced, as upon the plain scale, by the frequent application of the compasses.

One of the best plotting scales consists of two feather-edged rulers, one sliding along the other in a dovetailed groove, so that the two are always at right angles to each other. We shall describe this instrument more particularly when we come to speak of plotting, after describing the instruments used in surveying.

THE PANTAGRAPH.—The pantagraph consists of four rulers, A B, A C, D F, and E F, Fig. 45, made of stout brass. The two longer rulers, A B, and A C, are connected together by, and have a motion round a center at A. The two shorter rulers are connected in like manner with each other at F, and with the longer rulers at D and E, and, being equal in length to the portions AD and AE of the longer rulers, form with them an accurate parallelogram, ADFE, in every position of the instrument. Several ivory castors support the machine parallel to the paper, and allow it to move freely over it in all directions. The arms, AB and DF, are graduated and marked ½, ⅓, &c., and have each a sliding index, which can be fixed at any of the divisions by a milled-headed clamping screw, seen in the engraving. The sliding indices have each of them a tube, adapted either to slide on a pin rising from a heavy circular weight called the fulcrum, or to receive a sliding holder with a pencil or pen, or a blunt tracing point, as may be required.

When the instrument is correctly set, the tracing point, pencil, and fulcrum will be in one straight line, as shown by the dotted line in the figure. The motions of the tracing point and pencil are then, each, compounded of two circular motions, one about the fulcrum, and the other about the joints at the ends of the rulers upon which they are respectively placed. The radii of these motions form sides about equal angles of two similar triangles, of which the straight line BC, passing through the tracing point, pencil, and fulcrum, forms the third sides. The distances passed over by the tracing point and pencil, in consequence of either of these motions, have then the same ratio, and, therefore, the distances passed over in consequence of the combination of the two motions have also the same ratio, which is that indicated by the setting of the instrument.

Our engraving represents the pantagraph in the act of reducing a plan to a scale of half the original. For this purpose the sliding indices are first clamped at the divisions upon the arms marked ½; the tracing point is then fixed in a socket at C, over the original drawing; the pencil is next placed in the tube of the sliding index upon the ruler DF, over the paper to receive the copy; and the fulcrum is fixed to that at B, upon the ruler AB. The machine being now ready for use, if the tracing point at C be passed delicately and steadily over every line of the plan, a true copy, but of one-half the scale of the original, will be marked by the pencil on the paper beneath it. The fine thread represented as passing from the pencil quite round the instrument to the tracing point at C, enables the draughtsman at the tracing point to raise the pencil from the paper, whilst he passes the tracer from one part of the original to another, and thus to prethe thickness of the paper; and the paper beneath will receive corresponding marks, forming an exact copy, which is afterwards to be inked in.

Third Method.—The drawing is placed upon a large sheet of plate glass called a copying glass, and the paper to receive the copy placed over the drawing. The glass is then fixed in such a position as to have a strong light fall upon it from behind, and shine through it. By this means the original drawing becomes visible through the paper placed over it, and a copy can be made with precision and ease, without any risk of soiling or injuring the original.

To copy with nicety upon a reduced or enlarged Scale.—For this purpose we may have recourse to the method of squares, by which, with the aid of the proportional compasses, the most minute detail may be copied with great accuracy. This may, perhaps, be best shown by an example.

Let figure 47, in the engraving, represent a plan of an estate, which it is required to copy upon a reduced scale of one-half. The copy will therefore be half the length and half the breadth, and, consequently, will occupy but one-fourth of the space of the original. Our subject is a map of an estate, but the process would be precisely the same if it were an architectural, mechanical, or any other drawing.

1. Draw the lines F I and F G at right angles to each other. 2. From the point F towards I and G, set off any number of equal parts, as F *a*, *a b*, *b c*, &c., on the line F I, and F *i*, *i k*, *k l*, &c., on the line F G. 3. From the points on the line F I draw lines parallel to the other line F G, as *a a*, *b b*, *c c*, &c., and from the points on F G draw lines parallel to F I, as *i i*, *k k*, *l l*, &c., which being sufficiently extended towards G and I, the whole of the original drawing will be covered with a reticule of small but equally sized squares.

This done, draw upon the paper intended for the copy a similar set of squares, but having each side only one-half the length of the former, as is represented in figure 48. It will now be evident that, if the lines of the plan A B, B C, C D, &c., figure 1, be drawn in the corresponding squares of figure 2, a correct copy of the original will be produced, and of half the original scale. Commencing then at A, observe where, in the original, the angle A falls, which is towards the bottom of the square marked on the top *d e*. In the corresponding square, therefore, of the copy, and in the same proportion towards the left-hand side of it, which should be determined by the use of the proportional compasses, place the same point in the copy. From thence, finding by the proportional compasses the point on the bottom line of that square, where the curved line A F, crosses, which is about two-fifths from the left-hand corner towards the right, cross it similarly in the copy. Again, as it crosses the right-hand bottom corner of the second square below *d e*, describe it so in the copy; and by means of the proportional compasses find the points where it crosses the lines *f f* and *g g*, above the line *l l*, by taking the distances of such crossings from the nearest corner of a square in the original, between the large points of the proportional compasses, and with the small points at their opposite end, setting off the required crossing on the corresponding lines on the copy. Lastly, determine the place of the point B, in the third square below *g h*, on the top line; and a line drawn from A, in the copy, through these several points to B, will be a correct reduced copy of the original line. Proceed in like manner with every other line on the plan, and its various details, and you will have the plot or drawing, laid down to a small scale, yet bearing all the proportions in itself exactly as the original.

It may appear almost superfluous to remark, that the process of enlarging drawings by means of squares is a similar operation to the above, except that the points are to be determined in the smaller squares of the original, and transferred to the larger squares of the copy. The process of enlarging, under any circumstances, does not, however, admit of the same accuracy as that of reducing.

Having now completed the description of those instruments, applicable to the purposes of geometrical drawing, to the consideration of which we propose for the present to limit ourselves, in accordance with the plan of our work, we now propose to add thereto a description of *Coggeshall's Sliding Rule*, and then to conclude this part of our subject with some practical hints, which we think may prove acceptable to the commencing student.

COGGESHALL'S SLIDING RULE, FIG. 52.—Coggeshall's, or the Carpenter's Sliding Rule, is the instrument most commonly used for taking the dimensions, and finding the contents of timber. It consists of a rule one foot long, having on its face a groove throughout its entire length, in which a second rule of the same length slides smoothly. On the face of the rule are four logarithmic lines marked at one end A B C and D. The three lines A B C, are called *double* lines, because the figures from 1 to 10 are contained twice in the length of the rule, and are, in fact, repetitions of the logarithmic line of numbers already described. The fourth line D, is a *single* line numbered from 4 to 40, and is called the *Girt Line*, because the girt dimensions are estimated upon it in computing the contents of trees and timber. The lengths upon this line denote the logarithms of the squares of the numbers, from 4 to 40, placed against the several divisions; and enable us, as will be seen, to obtain approximately the content of a solid by a single operation.

The line C, is used with girt line D, and the two lines A and B, enable us to perform more readily all such operations, as have been already described as being performed by the logarithmic line of numbers with the aid of the compasses, the second line B, upon the slider, supplying the place of the compasses.

On the girt-line is a mark at the point 18·79, lettered G (*gallons*), which is the imperial gauge point, enabling us to compute contents in imperial gallons.

18·79 is the diameter of a cylindrical vessel to contain one gallon for each inch of depth. The gauge point for the old wine gallon was at 17·15, lettered W. G., and for the old ale gallon at 18·95, lettered A. G. These marks are consequently found upon rules constructed prior to January, 1826.

The back of the rule has a decimal scale of one foot divided into one hundred equal parts, by which dimensions are taken in decimals of a foot; and also a scale of inches, numbered from 1 to 12, which scale is continued on the slider and numbered from 12 to 24, so that, when the slider is pulled out, a two-feet rule is formed, divided into inches. The vacant spaces on the rule are filled up with various other scales and tables, which may be selected to suit the purposes of the various purchasers.

The method of notation on the rule, and the manner of estimating any number upon it, are the same as have already been fully explained, when treating of the line of logarithmic numbers.

Problem 1. *To multiply two numbers together.*—Set 1 on B, to the multiplier on A, and against the multiplicand on B, will be found the required product on A. *Example.*—To multiply 33 by 23. Set 1 on B, to 2·3 on A, and against 3·3 on B, will be found 7·59 on A, and 759 is therefore the product required.

Problem 2. *To divide one number by another.*—Set 1 on B, to the divisor on A, and against the dividend on A, will be found the required quotient on B.

Example.—To divide 759 by 23. Set 1 on B, to 2·3 on A, and against 75·9 on A, will be found 33 on B, which is the quotient required.

Problem 3. *To find a Fourth Proportional to three given numbers.*—Set the first term on B, to the second term on A, and against the third term on B, will be found the required fourth term on A. Or, against the first term on A, set the second term on B, and against the third term on A, will be found the required fourth term on B. *Example.*—To find a fourth proportional to the three numbers 3½, 11, and 14. Set 3½, or 3·5, on B to 11 on A, and against 14 on B will be found on A, 44, the fourth proportional required.

Problem 4. *To find a Third Proportional to two given numbers.*—This is the same problem as the preceding, repeating the second number for the third term of the proportion. *Example.*—To find a third proportional to the two numbers 3½ and 11. This is to find a fourth proportional to the three numbers 3½, 11 and 11. Set therefore 3½, or 3·5, on B, to 11 on A, and against 11 on B will be found on A, 34·6, the third proportional required.

Problem 5. *To square a given number. First method by means of the Lines* A *and* B.—Set 1 on B to the given number on A, and against the given number on B, will be found its square upon A. *Example.*—Required the square of 23. Set 1 on B to 23 on A, and against 23 on B, will be found its square 529 on A. *Second method, by means of the Lines* C *and* D.—If the number to be squared lie between 1 and 4, or 10 and 40, or 100 and 400, &c., so that its first significant digit is less than 4; set the 1 on C, to 10 on D, and against the digits on D, expressing the given number, will be found on C, the digits expressing the required square. Then, the square of 1 being 1, of 10, 100, of 100, 10,000, &c., and of ·1 being ·01, of ·01 being ·0001., &c., the digits upon C, must be estimated at the actual values, represented by them as numbered upon the scale, viz., 1, 2, &c., to 16, or at 100 times their values, from 100 up to 1600, or at 10,000 times their values from 10,000 up to 160,000, &c., or, again, at the $\frac{1}{100}$th part of these values from ·01 up to ·16, or at $\frac{1}{10000}$th part of these values from ·0001 up to ·0016, &c., according as the highest denomination in the number to be squared is units, or tens, or hundreds, &c., or again, tenths, hundredths, &c. *Example.*—Required the square of 23. The 10 on D, being set against the 1 C, against 23 on D, will be found 5·29 on C, and the highest denominations in 23 being tens, the square required is 529.

Also the squares of 2·3, 230, 2300, ·23, ·023 would be 5·29, 52,900, 5,290,000, ·0529, 000529, respectively, the highest denominations in the proposed numbers being respectively units, hundreds, thousands, tenths, and hundredths. *Third method, by means of the Lines* C *and* D.—If the number to be squared lie between 4 and 10, or 40 and 100, or 400 and 1000, &c., so that its first significant digit is not less than 4; set the 100 on C against the 10 on D, and against the digits on D, expressing the given number, will be found on C the digits expressing the required square. *Example.*—Required the square of 51. The 100 on C being set against the 10 on D, against 5·1 on D will be found 26 on C, and, the highest denomination in 51 being tens, the square required is 2600.

Problem 6. *To extract the Square Root of a given Number.*—This problem being the converse of the preceding, set the rule in the same manner, with the 1 on C against the 10 on D, if the given square be between 1 and 16, or 100 and 1600, or 10,000 and 160,000, &c., or again between ·01 and ·16, or ·0001 and ·0016, &c., and with the 100 on C against the 10 on D, if the given square be between 16 and 100, or 1600 and 10,000, &c., or again between ·16 and 1, or ·0016 and ·01, &c.; and then against the given number on C will stand its square root on D. *Example* 1.—Required the square root of 529. The given number being between 100 and 1600, set the 1 on C against the 10 on D, and against 5·29 on C will be found 23 on C, the square root required. *Example* 2.—Required the square root 2601. The given number being between 1600 and 10,000, set the 100 on C against the 10 on D, and against 26 on C will be found 5·1 on D, and 51 is therefore the root sought.

Problem 7. *To find a mean Proportional between two given Numbers.*—Set one of the numbers upon C to the same number on D, and against the other number on C will be found upon D the mean proportional required. *Example.*—Required a mean proportional between 4 and 49. Set 4 on C to 4 on D, and against 49 on C will be found on D 14, the mean proportional required.

If one number exceed the other so much that they cannot both be taken off from the line C, the $\frac{1}{100}$th part of the larger may be taken, and the mean proportional then found, multiplied by 10, will give the mean proportional required. Also if the second number on C be situated beyond the scale D, the $\frac{1}{100}$th part of such second number may be substituted for it, and the result multiplied by 10, or 100 times such number may be taken, and the result divided by 10, or again such second number may be multiplied or divided by 4, 9, or any square number, and the result divided or multiplied by 2, 3, or the square root of this number; or, again, the numbers may be both multiplied and divided by any of the same number, and the result divided or multiplied also by the same number, and, in each case, the required mean proportional will be correctly determined.

Problem 8. *To find the Area of a Board or Plank. First Method.*—Set 12 on B to the mean breadth in inches on A, and against the length in feet on B will be found upon A the required area in feet and decimals of a foot. If the plank taper regularly, the mean breadth is half the sum of the extreme breadths; but, if the plank be irregular, several breadths should be measured at equal distances from each other, and their sum divided by their number may be taken as the mean breadth. In the latter case, however, greater accuracy would be obtained by finding separately the areas of portions of the plank, and adding them together for the whole area, or by the following. *Second Method.*—Take the measure in inches of several breadths at equal distances from each other, and add together half the two extreme breadths, and the sum of all the intermediate breadths. Set 12 on B to the sum thus found upon A, and against the distance in feet, at which the breadths have been measured, upon B will be found upon A the required area in feet and decimals of a foot. *Example* 1.—A board, 15 feet long, being 14 inches broad at one end, and 8 inches broad at the other, required its area. The mean breadth is 11 inches, half the sum of 8 and 14. Set, then, 12 on B against 11 on A, and against 15 on B will be found upon A 13·75 or 13¾ feet, the area required. *Example* 2.—An irregular board, 18 feet long, being 7 inches broad at one extremity, 11 inches broad at the other, and the intermediate breadths at each 3 feet of the length being 13 inches, 25 inches, 23 inches, 32 inches, and 22 inches, required its area. By the first method, the sum of the seven breadths divided by 7, gives 19 inches for the mean breadth; and, setting 12 on B against 19 on A, against 18 on B will be found upon A 28·5 or 28½ feet, the area required. By the second method, half of the two extreme breadths added to the intermediate breadths, gives the sum, 123 inches; and setting 12 on B against 123 upon A, against 3 on B will be found upon A 30¾, the area required, a more accurate result than the preceding.

Problem 9. *To find the solid Content of squared or four-sided Timber, of the same size throughout its entire Length. First Method.*—Multiply the breadth by the thickness, and their product again by the length (Problem 1), and the result will be the content required. *Second Method.*—Set the length on C against 12 on D, and against the quarter girt, measured in inches, on D, will be found the approximated content on C in cubic feet: or set the length on C against 10 on D, and against the quarter girt, measured in tenths of feet, on D will be found the approximate content on C. The approximate content thus found is greater than the true content, and the correction to be subtracted to leave the true content is given in the following Table:—

Fraction of breadth equal to excess of breadth over thickness.	Excess in inches for each 12 inches of breadth.	Fractional portion of approximate content to be subtracted.	Per-centage of approximate content to be subtracted.
$\frac{1}{2}$ breadth	6 inches.	$\frac{1}{9}$ app. cont.	11 per cent.
$\frac{1}{3}$ "	4 "	$\frac{1}{25}$ "	4 "
$\frac{1}{4}$ "	3 "	$\frac{1}{49}$ "	2 "
$\frac{1}{5}$ "	$2\frac{4}{10}$ "	$\frac{1}{81}$ "	$1\frac{1}{4}$ or 1·23 "
$\frac{1}{6}$ "	2 "	$\frac{1}{121}$ "	$\frac{5}{6}$ or ·83 "
$\frac{1}{7}$ "	$1\frac{5}{7}$ "	$\frac{1}{169}$ "	$\frac{10}{17}$ or ·59 "
$\frac{1}{8}$ "	$1\frac{1}{2}$ "	$\frac{1}{225}$ "	$\frac{4}{9}$ or ·44 "
$\frac{1}{9}$ "	$1\frac{1}{3}$ "	$\frac{1}{289}$ "	$\frac{1}{3}$ or ·35 "
$\frac{1}{10}$ "	$1\frac{2}{10}$ "	$\frac{1}{361}$ "	$\frac{2}{7}$ or ·28 "
$\frac{1}{11}$ "	$1\frac{1}{11}$ "	$\frac{1}{441}$ "	$\frac{5}{22}$ or ·23 "
$\frac{1}{12}$ "	1	$\frac{1}{529}$ "	$\frac{10}{52}$ or ·19 "

The fractional portion of the approximate contents in column 3 may be found by dividing the approximate contents by the denomination of the fractions. (Problem 2.)

If the excess of the breadth over the thickness be compared with the quarter girt, the correction has to the approximate content the duplicate ratio of half the excess to the quarter girt, as shown in the following Table:—

Fraction of quarter girt equal to half the excess of breadth over thickness.	Half the excess of breadth over thickness for each 12 inches of quarter girt.	Correction to be subtracted.	
		Fractional portion of approximate content.	Per-centage of approximate content.
$\frac{1}{2}$ qr. girt.	6 inches.	$\frac{1}{4}$ approximate content.	25 per cent.
	5 "	$\frac{1}{6}$ nearly or $\frac{25}{144}$ "	$17\frac{1}{3}$ or 17·36 "
$\frac{1}{3}$ "	4 "	$\frac{1}{9}$ "	11 "
$\frac{1}{4}$ "	3 "	$\frac{1}{16}$ "	$6\frac{1}{4}$ or 6·25 "
$\frac{1}{5}$ "	$2\frac{4}{10}$ "	$\frac{1}{25}$ "	4 "
$\frac{1}{6}$ "	2 "	$\frac{1}{36}$ "	$2\frac{7}{9}$ or 2·78 "
$\frac{1}{7}$ "	$1\frac{5}{7}$ "	$\frac{1}{49}$ "	2·04 "
$\frac{1}{8}$ "	$1\frac{1}{2}$ "	$\frac{1}{64}$ "	$1\frac{3}{5}$ or 1·56 "
$\frac{1}{9}$ "	$1\frac{1}{3}$ "	$\frac{1}{81}$ "	$1\frac{1}{4}$ or 1·23 "
$\frac{1}{10}$ "	$1\frac{2}{10}$ "	$\frac{1}{100}$ "	1 "
$\frac{1}{11}$ "	$1\frac{1}{11}$ "	$\frac{1}{121}$ "	$\frac{4}{5}$ or ·83 "
$\frac{1}{12}$ "	1 "	$\frac{1}{144}$ "	$\frac{7}{10}$ or ·69 "
$\frac{1}{13}$ "	$\frac{12}{13}$ "	$\frac{1}{169}$ "	$\frac{4}{7}$ or ·6 "
$\frac{1}{14}$ "	$\frac{6}{7}$ "	$\frac{1}{196}$ "	$\frac{1}{2}$ or ·51 "
$\frac{1}{15}$ '	$\frac{8}{10}$ "	$\frac{1}{225}$ "	$\frac{4}{9}$ or ·44 "

The correction may also be found as follows:—Set the length upon C, against 12, upon D, and against half the excess of the breadth over the thickness upon D, will be found upon C, the required correction in cubic feet.

As the error of the result obtained with the rule may amount to the $\frac{1}{200}$th part of the whole, the correction given above may always be neglected, whenever the excess of the breadth over the thickness does not exceed the $\frac{1}{8}$th part of the breadth, or $1\frac{1}{2}$ inch for each 12 inches of breadth, and the result may be depended upon to as great an accuracy as can be obtained by the rule. When, however, the excess is more than two inches for each 12 inches of breadth, either the correction should be applied or the first method be used.

Example 1.—Required the content of a piece of timber 10 inches broad, 8 inches thick, and 18 feet long.

Since $\frac{10}{12} \times \frac{8}{12} = \frac{80}{144}$, set 80 on B, against 144 on A, and against 18 on A, will be found 10 on B, and the content required is 10 cubic feet. *Example* 2.—Required the content of a piece of timber 15 inches broad, 10 inches thick, and 24 feet long. Set 24 on C, against 12 on D, and against 12·5, or $12\frac{1}{2}$, the quarter girt on D, will be found on C, 26·04, the approximate content. The excess of 15 over 10 being

$\frac{1}{3}$d of 15, our table shows the required correction to $\frac{1}{25}$th of 26·04. Set then 25 on B, against 26·04 on A, and against 1 on B, will be found 1·04 on A, which subtracted from 26·04 cubic feet leaves 25 cubic feet, the true content.

Problem 10. *To find the Content of a piece of square timber, which tapers from end to end.*—Set the length in feet upon C, against 12 upon D, and against half the sum in inches of the quarter girts at the two ends upon D, will be found a content in cubic feet upon C. Again, set one-third of the length in feet upon C, against 12, upon D, and against half the difference, in inches, of the quarter girts at the two ends upon D, will be found a second content in cubic feet upon C. Add together the two contents thus found for the content required. If the breadth exceed the thickness considerably, the same part of the result must be subtracted, as in Problem 9.

Example.—The quarter girts at the ends of a piece of timber 21 feet long, being 22 inches and 10 inches, respectively, and the breadth not much exceeding the thickness, required the content. Set 21 upon C, against 12 on D, and against 16 upon D, will be found 37$\frac{1}{3}$ or 37·3 upon C. Again, set 7 upon C, against 12 upon D, and against 6, upon D, will be 1$\frac{3}{4}$ or 1·75 upon C. The sum of 37$\frac{1}{3}$ cubic feet and 1$\frac{3}{4}$ cubic feet, is then 39$\frac{1}{12}$ or 39·1 cubic feet, the whole content required.

Problem 11. *To find the Content of a round piece of timber of the same size throughout its entire length.*—Set the length in feet upon C, against 10·63 (a mark is placed upon the rule at this point, 10·63 being the quarter girt in inches of the circle, whose area is a square foot,) upon D, and against the quarter girt in inches upon D, will be found the content upon C. *Example.*—Required the content of a round piece of timber 32 feet long, the quarter girt being 11 inches. Set 32 upon C, against 10·63, upon D, and against 11, upon D, will be found upon C, 34·25 or 32$\frac{1}{4}$, the content required.

Problem 12. *To find the Content of a round piece of timber, which tapers from end to end.*—Set the length in feet upon C, against 10·63, upon D, and against half the sum in inches of the quarter girts at the two ends upon D, will be found a content in cubic feet upon C. Again, set one-third of the length in feet upon C, against 10·63 upon D, and against half the difference in inches of the quarter girts at the two ends upon D, will be found a second content in cubic feet upon C. Add together the two contents thus found for the content required.

NOTE.—In buying rough or unsquared timber, an allowance of about $\frac{1}{12}$th should be made for the bark. A further allowance should also be made for the loss in squaring down the tree to make useful shaped timber. The whole amount of timber to be taken off to make a square piece from a round piece of timber will be 36 per cent., or more than a third of the whole. The timber so taken off must not, however, be considered completely valueless. If the length upon C be set against 12 upon D, instead of upon 10·63 in the two preceding problems, this will be equivalent to an allowance of about 21½ per cent., which may be considered a just allowance.

Example 1.—A piece of round tapering timber measures 23 feet in length, the quarter girt at the larger end is 23½ inches, and at the smaller end the quarter girt is 15½ inches. Required the true content. Set 23 upon C against 10·63 upon D, and against 19½, 19·5, or upon D will be found 77·5 upon C. Again, set 7⅔ or 7·66 upon C against 10·63 upon D, and against 8 upon D will be found 4·3 upon C. Then the sum of 77·5 cubic feet and 4·3 cubic feet is 81·8 cubic feet, the content required. *Example* 2.—Required the content of a piece of unsquared timber of the same dimensions as in the preceding example, making allowance of 21½ per cent. for loss in squaring down into a useful shape. Set 23 upon C against 12 upon D, and against 19½, or 19·5, upon D will be found 60·75 upon C. Again, set 7⅔ or 7·66 upon C against 12 upon D, and against 8 upon D will be found 3·4 upon C. Then the sum of 60·75 cubic feet and 3·4 cubic feet is 64.15 cubic feet, the content required.

Problem 13. *To find the Content of a Cylindrical Vessel in Gallons.*—Set the length of the cylinder in inches upon C against the gauge mark at 18·79, marked G, upon D, and against the diameter of the cylinder upon D, will be found the required content in gallons upon C. If the number of inches in the diameter lie beyond C, or if this number be greater than 40, so as not to be contained upon D, the $\frac{1}{10}$th part, or any part that may be convenient, of the number of inches in the diameter may be taken, and the result thus obtained, multiplied by 100, or the square of the divisor made use of, will give the content required. *Example.*—A circular vat 5 feet in diameter being filled to the depth of four feet, required the quantity of liquor in it. Set 48 upon C against the gauge mark at 18·79 upon D, and against 6, the $\frac{1}{10}$th part of the diameter in inches, upon D will be found upon C 4·9; and consequently 4·9 × 100 or 490 gallons, is the quantity of liquor in the vat.

PRACTICAL HINTS ON THE MANAGEMENT OF DRAWING PAPER.

THE first thing to be done preparatory to the commencement of a drawing, is to stretch the paper evenly upon the smooth and flat surface of a drawing board. The edges of the paper should first be cut straight, and, as nearly as possible, at right angles with each other; also the sheet should be so much larger than the intended drawing and its margin, so as to admit of being afterwards cut from the board, leaving the border by which it is attached thereto by glue or paste, as we shall next explain.

The paper must first be thoroughly and equally damped with a sponge and clear water, on the opposite side from that on which the drawing is to be made. When the paper absorbs the water, which may be seen by the wetted side becoming dim, as its surface is viewed slantwise against the light, it is to be laid on the drawing board with the wetted side downwards, and placed so that its edges may be nearly parallel with those of the board; otherwise, in using a T square, an inconvenience may be experienced. This done, lay a straight flat ruler on the paper, with its edge parallel to, and about half an inch from, one of its edges. The ruler must now be held firm, while the said projecting half inch of paper is turned up along its edge; then, a piece of solid glue (common glue will answer the purpose), having its edge partially dissolved by holding it in boiling water for a few seconds, must be passed once or twice along the turned edge of the paper, after which, this glued border must be again laid flat by sliding the rule over it, and, the ruler being pressed down upon it, the edge of the paper will adhere to the board. If sufficient glue has been applied, the ruler may be removed directly, and the edge finally rubbed down by an ivory book-knife, or any clean polished substance at hand, which will then firmly cement the paper to the board. Another but *adjoining* edge of the paper must, next, be acted upon in like manner, and then the remaining edges in succession; we say the adjoining edges, because we have occasionally observed that, when the opposite and parallel edges have been laid down first, without continuing the process progressively round the board, a greater degree of care is required to prevent undulations in the paper as it dries.

Sometimes strong paste is used instead of glue; but, as this takes a longer time to set, it is usual to wet the paper also on the upper surface to within an inch of the paste mark, care being taken not to rub or injure the surface in the process. The wetting of the paper in either case is for the purpose of expanding it; and the edges, being fixed to the board in its enlarged state, act as stretchers upon the paper, while it contracts in drying, which it should be allowed to do gradually. All creases or undulations by this means disappear from the surface, and it forms a smooth plane to receive the drawing.

TABLE OF DIMENSIONS OF DRAWING PAPER.

Demy,	20	Inches	by	15¼	inches.	Columbier,	35	Inches	by	23½	Inches.
Medium,	22¾	“		17½	“	Atlas,	34	“		26	“
Royal,	24	“		19¼	“	D'ble Elephant,	40	“		7	“
Super Royal,	27¼	“		19¼	“	Antiquarian,	53	“		31	“
Imperial,	30	“		22	“	Emperor,	60	“		48	“
Elephant,	28	“		23	“						

MOUNTING PAPER AND DRAWINGS, VARNISHING, ETC.

In mounting paper upon canvas, the latter should be well stretched upon a smooth flat surface, being damped for that purpose, and its edges glued down as was recommended in stretching drawing paper. Then with a brush spread strong paste upon the canvas, beating it in till the grain of the canvas be all filled up; for this, when dry, will prevent the canvas from shrinking when subsequently removed; and, having cut the edges of the paper straight, paste one side of every sheet, and lay them upon the canvas, sheet by sheet, overlapping each other a small quantity. If the drawing paper is strong, it is best to let every sheet lie five or six minutes after the paste is put on it; for, as the paste soaks in, the paper will stretch, and may be better spread smooth upon the canvas; whereas, if it be laid on before the paste has moistened the paper, it will stretch afterwards and rise in blisters when laid upon the canvas. The paper should not be cut off from its extended position till thoroughly dry; and the drying should not be hastened, but gradually take place in a dry room, if time permit; if not, the paper may be exposed to the sun, unless in the winter season, when the help of a fire is necessary, care being had that it is not placed too near a scorching heat.

In joining two sheets of paper together by overlapping, it is necessary, in order to make a neat joint, to feather-edge each sheet; this is done by carefully cutting with a knife, half way through the paper near the edges, and on the sides, which are to overlap each other; then strip off a feather-edged slip from each, which, being done dexterously, the edges will form a very neat and efficient joint when put together.

The following method of mounting and varnishing drawings or prints was communicated some years ago by Mr. Peacock, an artist of Dublin. Stretch a piece of linen on a frame, to which give a coat of isinglass or common size. Paste the back of the drawing, leave it to soak, and then lay it on the linen. When dry, give it at least four coats of well made isinglass size, allowing it to dry between each coat. Take Canada balsam diluted with the best oil of turpentine, and with a clean brush give it a full flowing coat.

GENERAL RULES APPLICABLE IN ALL GEOMETRICAL CONSTRUCTIONS.

In selecting black-lead pencils for use, it may be remarked that they ought not to be very soft, nor so hard that their traces cannot be easily erased by the India rubber. Great care should be taken, in the penciling, that an accurate outline be drawn; the pencil marks should be distinct yet not heavy, and the use of the rubber should be avoided as much as possible, for its frequent application ruffles the surface of the paper, and will destroy the good effect of shading or coloring, if any is afterwards to be applied.

The following seven useful rules are taken from Mr. Thomas Bradley's valuable work on Practical Geometry:—

1. Arcs of circles, or right lines by which an important point is to be found, should never intersect each other very obliquely, or at an angle of less than 15 or 20 degrees; and, if this cannot be avoided, some other proceeding should be had recourse to, to define the point more precisely.

2. When one arc of a circle is described, and a point in it is to be determined by the intersection of another arc, this latter need not be drawn at all, but only the point marked off on the first, as it is always desirable to avoid the drawing of

unnecessary lines. The same observation applies to a point to be determined on one straight line by the intersection of another.

3. Whenever the compasses can be used in any part of a construction, or to construct the whole problem, they are to be preferred to the rule, unless the process is much more circuitous, or unless the first rule (above) forbids.

4. A right line should never be obtained by the prolongation of a very short one, unless some point in that prolongation is first found by some other means, especially in any essential part of a problem.

5. The larger the scale on which any problem, or any part of one, is constructed, the less liable is the result to error; hence all angles should be set off on the largest circles which circumstances will admit of being described, and the largest radius should be taken to describe the arcs by which a point is to be found through which a right line is to be drawn; and the greater attention is to be paid to this rule, in proportion as that step of the problem under consideration is conducive to the correctness of the final result.

6. All lines, perpendicular or parallel to another, should be drawn long enough at once, to obviate the necessity of producing them.

7. Whenever a line is required to be drawn to a point, in order to insure the coincidence of them, it is better to commence the line from the point; and if the line is to pass through two points, before drawing it the pencil should be moved along the rule, so as to ascertain whether the line will, when drawn, pass through them both. Thus, if several radii to a circle were required to pass through any number of points respectively, the lines should be begun from the center of the circle; any error being more obvious when several lines meet in a point.

MENSURATION.

MENSURATION OF SUPERFICES.

To find the area of a four sided figure, whether it be a square, a parallelogram, a rhombus, or a rhomboid.

Rule.—Multiply the length by the breadth, or height, and the product will be the area.

To find the area of a right angled triangle.

Rule.—Multiply the base by half the perpendicular, and the product will be the area.

To find the area of an acute angled triangle.

Rule.—Multiply the length of the longest side by half the perpendicular let fall upon it from the opposite angle, and the product will be the area.

Having the base and perpendicular of a right angled triangle, to find the Hypothenuse.

Rule.—Add together the squares of the base and perpendicular, and the square root of the sum will be the hypothenuse.

Example.—If the span of a roof be 24 feet, and the height 5 feet, what would be the length of the rafters? This is resolved into two equal and similar triangles, the base of each being half the span or 12 feet; and the perpendicular equal to the height or 5 feet.

```
     5          12
     5          12
    ——         ———
    25         144 Square of base.
                25 Square of perpendicular.
               ———
               1·69 (13 feet the length of the rafter.
               1
               ———
           23) 69
               69
```

When the Hypothenuse and one leg are given, to find the other leg.

Rule.—Subtract the square of the leg from the square of the hypothenuse, and the square root of the remainder will give the other leg.

Example.—In a roof whose rafters are 26 feet long, and the height 10 feet, what is the span?

10 26
10 26
100 Square of height. 156 Square of rafters.
52
676
100
5·76 (24 feet half the span.
4
44)176
176

Example 2.—Wanted to prop a building with raking shores, at the height of 25 feet from the ground, the shores being 30 feet long; how far from the base of the building must they be placed?

30 25
30 25
900 Square of shores. 125
625 50
2·75(16·58, or 16 ft. 5⅞ in. 625 Square of height.
1 from the building.
26)175
156
325) 1900
1625
3308) 27500
26464

To find the Length of a Cylindrical Helix.

Rule.—Multiply the circumference of the base, by the number of revolutions, to the square of the product, add the square of the height of the spiral, and the square root of the sum will be the height of the spiral; for the helix, if unwound off the cylinder and stretched out, would be a right angled triangle, the length of whose base is the number of revolutions, and the height that of the spiral.

The Chord and Versed Sine of the Arc of a Circle being given; to find the diameter.

Rule 1.—Divide the sum of the squares of the sine and versed sine, by the versed sine itself; and the quotient is the diameter.

Rule 2.—Divide the square of half the chord by the versed sine, and the quotient is the diameter wanting the versed sine; to which, add the versed sine, and the product is the diameter.

Example.—Required the length of a rod or wire that will from a center strike a segmental arch, whose chord or span is 28 feet, and the rise 4 feet.

$$\begin{array}{r l} 14 & \text{half the chord.} \\ 14 & \\ \hline 56 & \\ 14 & \\ \hline 4)196 & \\ \hline 49 & \\ 4 & \\ \hline \end{array}$$

It will take a rod 53 feet long.

To find the Area of a Circular Ring, or Space included between two concentric circles.

Rule.—Find the areas of the two circles separately; then the difference between them will be the area of the ring.

To find the Circumference of an Ellipse.

Rule.—Square the two axes or diameters, and multiply the square root of half that sum by 3·1416; the product will be the circumference nearly.

To find the Area of an Ellipse.

Rule.—Multiply the transverse diameter by the conjugate, and the product by ·7854; this last product will be the area.

To find the Area of a Trapezium.

Rule.—Draw a diagonal line from the two longest opposite angles, which will divide the figure into two triangles, which measure as before.

To find the Circumference of a Circle, when the Diameter is given; or the Diameter, when the Circumference is given.

Rule.—Multiply the diameter by 3·1416, and the product will be the circumference; or divide the circumference by 3·1416, and the quotient will be the diameter.

To find the Area of a Circle.

Rule 1.—Multiply the square of the diameter by ·7854, or the square of the circumference by ·07958; the product, in either case, will be the area.

Rule 2.—Multiply the circumference by the diameter, and divide the product by 4.

Rule 3.—As 14 is to 11, so is the square of the diameter to the area; or as 88 is to 7, so is the square of the circumference to the area.

To find the Length of any Arc of a Circle.

Rule 1.—From 8 times the chord of half the arc, subtract the chord of the whole arc; one-third of the remainder will be the length of the arc nearly.

Rule 2.—As 180 is to the number of degrees in the arc, so is 3·1416 times the radius to its length; or, as 3 is to the number of degrees in the arc, so is ·05236 times the radius to its length.

To find the Area of a Sector of a Circle.

Rule 1.—Multiply the length of the arc, by half the length of the radius; the product will be the area.

Rule 2.—As 360 degrees is to the number of degrees in the arc of the sector, so is the area of the circle to the area of the sector.

To find the Area of a Segment of a Circle.

Rule.—To the chord of the whole arc, add the chord of half the arc and one-third of it more. Then multiply the sum by the versed sine, or height of the segment, and four-tenths of the product will be the area of the segment.

To find the Area of an Elliptic Segment.

Rule.—Divide the height of the segment by the axis of which it is a part, and find in the table of circular segments, a segment having the same versed sine as this quotient. Then multiply the segment thus found, and the two axes of the ellipse continually together, and the product will give the area required.

OF SOLIDS.

To find the Solidity of a Cube or Parallelopipedon.

Rule.—Multiply the side of the cube by itself, and that product by the side or length; this last product will be the solidity.

To find the Solidity of a Cylinder.

Rule.—Multiply the area of the base, by the height of the cylinder, and the product will be the solid content.

To find the Convex Surface of a Cylinder.

Rule.—Multiply the circumference by the length of the cylinder, and the product will be the convex surface required.

To find the Convex Surface of a Right Cone or Pyramid.

Rule.—Multiply the circumference of the base by the slant height or length of the side, and half the product will be the surface.

To find the Convex Surface of a frustrum of a Right Cone or Pyramid.

Rule.—Multiply the sum of the perimeters of the two ends by the slant height or side of the frustum, and half the product will be the surface required.

To find the Solidity of a Cone or Pyramid.

Rule.—Multiply the area of the base by the height, and one-third of the product will be the content.

To find the Solidity of the Frustum of a Cone.

Rule.—Divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters; this quotient multiplied by ·7854, and again by one-third of the height, will give the solidity.

To find the Solidity of a Wedge.

Rule.—To the length of the edge of the wedge, add twice the length of the back; multiply this sum by the height of the wedge, and then by the breadth of the back; one-sixth of the product will be the solid contents.

To find the Solidity of a Prismoid.

Rule.—Add into one sum the areas of the two ends, and four times the middle section parallel to them; then, this sum multiplied by one-sixth of the height, will give the contents.

NOTE.—The length of the middle section is equal to half the sum of the lengths of the two ends, and its breadth is equal to half the sum of the breadths of the two ends.

To find the Convex Surface of a Sphere or Globe.

Rule.—Multiply the diameter of the sphere by its circumference; or, multiply 3·1416 by the square of the diameter; the product in either way will be the convex surface required.

NOTE.—The convex surface of any zone or segment may be found in like manner, by multiplying its height by the whole circumference of the sphere.

To find the Solidity of a Sphere or Globe.

Rule.—Multiply the square of the diameter by ·7854, and this product again by the diameter, and two-thirds of this last product will be the solidity; or, multiply the cube of the axis by ·5236; the product will be the solidity.

To find the Solidity of a Spherical Segment.

Rule.—To three times the square of the radius of its base, add the square of its height; then multiply the sum by the height, and the product by ·5236.

To find the Solidity of a Spherical Zone or Frustum.

Rule.—To the sum of the squares of the radius of each end, add one-third of the square of the height of the zone; this sum multiplied by the said height, and the product by 1·5708, will give the solidity.

To find the Solidity of a Spheroid.

Rule.—Multiply the square of the revolving axis, by the fixed axis; the product, multiplied by ·5236, will give the contents.

To find the Solidity of a Dome, less than a Hemisphere upon a square plan, when its axal sections parallel to the sides are circles.

Rule.—From twice the diameter of the vertical section multiplied into the square of the altitude, subtract the third part of 4 times the cube of the altitude, and the remainder is the solidity of the square segmental dome.

A CONCISE PRACTICAL METHOD OF FINDING THE NUMBER OF CUBIC FEET AND INCHES CONTAINED IN TIMBER AND OTHER MATERIALS.

If the length be given in feet and inches, and the section, or end, in inches, multiply the sides of the section by each other, and divide by 12. Also divide the length by 12; multiply these two dimensions by each other duodecimally, and the product will be the contents in cubic feet and inches.

Example.—Find the number of cubic feet, in a piece of timber 28 feet long, 11 inches wide, and 3 inches thick.

in.
$\left.\begin{matrix}11\\3\end{matrix}\right\}$ the sides.
12(33
2·9

ft.
12)28 the length,
Multiply 2.4
by 2·9
4·8
1·9·0
6·5 gives 6 ft. 5 in. the solidity.

Example 2.—Find the cubic contents of 4 quarters or studs, each 12 feet 6 inches long, and 6 inches wide, by 2½ inches thick.

$\left.\begin{matrix}2\frac{1}{2}\\6\end{matrix}\right\}$ the sides.
12)15
1·3

12·6 the length.
4 the number of pieces.
12)50·0
Multiply 4·2
by 1·3
4·2
1·0·6
5·2·6 gives 5 ft. 2 in. and 6 parts the solidity.

larities; consequently, single joisted floors of long bearing can be used only in inferior buildings.

When it is desirable to have a perfect ceiling, a double floor is used; and when the bearing is long, a framed floor becomes the most convenient. The following experiment was made on the comparative strength of framed and single joisted floors, by Professor Robinson:

Two models were made 18 inches square; one consisted of single joists, the other framed with girders, binding, bridging, and ceiling joists; the single joists of the one contained the same quantity of timber with the girders alone of the other. They were placed in a wooden trunk 18 inches square within, with a strong projection on the inside for the floors to rest on, and small shot was gradually poured over. The single joisted floor broke down with 487 pounds; the framed floor, with 327 pounds. The difference would not be quite so much on a large scale, because the girders would not be so much weakened by mortices. This is not the only case where apparent strength has turned out to be real weakness; and shows how necessary it is to distinguish those parts which really support a load, from those which only appear to do so.

OF SINGLE JOISTED FLOORS.

In order to make a strong floor, with a small quantity of timber, the joists should be thin and deep; but a certain degree of thickness is necessary, for the purpose of nailing the boards; and two inches is perhaps as thin as the joists ought to be made.

To find the depth of a Joist, the length of bearing and thickness being given.

Rule.—Divide the square of the length in feet, by the thickness in inches; and the cube root of the quotient multiplied by 2·2 for pine, or 2·3 for oak, will give the depth in inches.

The two joists which support the trimmers to fireplaces, &c., called *trimming joists*, should, as well as the trimmers, be stronger than the common joists. In general, it will be sufficient to add one-eighth of an inch to the thickness of the trimming joists, for each joist supported by the trimmer. Thus, if the thickness of the common joists be 2 inches, and a trimmer supports 4 joists, then add 4 eighths, or half an inch, that is, make the trimming joists each 2½ inches thick.

When the bearing exceeds 10 feet, single joisting should be strutted between the joists to prevent them from turning or twisting sideways, and also to stiffen the floor; when the bearing exceeds 15 feet, two rows of struts will be necessary, and so on, adding another row of struts for each increase of 5 feet in bearing. Herringbone strutting is found to answer best, and the well fitting and nailing of these between the joists in straight rows across the whole room, is an essential part in making a good ceiling.

For common purposes, single joisting may be used to any extent for which timber can be got deep enough; but where it is desirable to have a perfect ceiling, the bearing should not exceed 18 feet.

OF FRAMED FLOORS.

The girders are the chief support of a framed floor, and their depth is often limited by the size of the timber; therefore, the method of finding the scantling may be divided into two cases.

Case 1.—*To find the depth of a girder, when the length of bearing and thickness of the girder are given.*

Rule.—Divide the square of the length in feet, by the thickness in inches; and the cube root of the quotient multiplied by 4·2 for pine, or by 4·34 for oak, will give the depth required in inches.

Case 2.—*To find the thickness when the length of bearing and depth are given.*

Rule.—Divide the square of the length in feet, by the cube of the depth in inches, and the quotient multiplied by 74 for pine, or by 82 for oak, will give the depth in inches.

In these rules the girders are supposed to be 10 feet apart, and this distance should never be exceeded; but should the distance apart be more or less than 10 feet, the thickness of the girder should be made proportionate thereto.

Girders should always, for long bearings, be made as deep as they can be got; an inch or two taken from the height of a room, is of little consequence compared with a ceiling disfigured with cracks; besides the inconvenience of not being able to move in the rooms above, without shaking every thing in them.

When the breadth of a girder is considerable, it is often sawn down the middle and bolted together, with the sawn sides outwards. This is an excellent method, as it not only gives an opportunity of examining the center of the log, which from large trees is often in a state of decay; but also reduces the timber to a smaller scantling, by which means it dries sooner, and is less liable to rot. The slips put between the halves, or flitches, should be thick enough to allow the air to circulate freely between them. It is generally imagined that it strengthens a girder to cut it down, reverse it, and bolt it together again; it is, in fact, weakened by the operation; but the method is here recommended, for the reasons above stated. We find that Vitruvius, the oldest author on architecture extant, directs a space of two fingers' breadth to be left between the beams for forming the architrave over columns, in order that the air may circulate between, and prevent decay.

Every one must have observed that decay begins in the first place at the joints, and other parts where the pieces are neither perfectly close, nor yet sufficiently open to allow any dampness to evaporate.

When the bearing exceeds 25 feet, it is difficult to obtain timber large enough for girders, and it is usual, in such cases, to truss them. The methods adopted for that purpose have generally the appearance of much ingenuity; but, in reality, they are of very little use. If a girder be trussed with oak, or other hard wood, all the strength that can possibly be gained by such a truss, consists merely in the difference between the compressibility of these materials and of pine, which is very small indeed; and unless the truss be extremely well fitted at the abutments, it would be much stronger without trussing. All the apparent stiffness obtained by trussing a beam, is produced by forcing the abutments, or, in other words, by cambering the beam. This forcing cripples and injures the natural elasticity of the timber, and the continual spring from the motion of the floor, upon parts already crippled, it may easily be conceived, will so far destroy them, as to render the truss a useless burden upon the beam. This fact has been long known to scientific builders, which has caused them to seek for a remedy in iron trusses; but this method is as bad as the former, unless there be an iron tie as an abutment to the truss, for the failure of a truss is occasioned by the great compression, or force, exerted upon a small surface of the timber at the abutments.

Though the usual mode of trussing girders cannot be relied on, nor, indeed any other timber truss that can be made within the depth of the beam; yet, by adding to the depth, there are several methods that may be applied with success in extending the bearing of timber girders; but where the depth is limited, and the bearing considerable, iron must be employed, and the best mode of doing this would be to make the girders of cast iron, each in one piece, if the bearing should not be too long for a casting, and in two pieces when that is the case. These cast iron girders are simple, and cheaper than any kind of iron framing of the same strength, that is, when they are properly contrived, so as to make the most of the material; but it often happens that large foundries are not near, and consequently iron girders would be very expensive; at any rate, it is not proper to omit showing how they may be done without, where there is the means of increasing the depth of the floor, which may generally be done without inconvenience.

The principle of constructing girders, of any depth, is the same as that of building beams, and, when properly conducted, is as strong as any truss that can be made of the same depth. A very good method consists in bending a piece of timber about half the depth of the lower beam into a curve, by inserting a piece cut into the curved form like a center between the two, and securing the upper piece from springing back by means of strong flat iron hoops, which may be driven on from the ends until they become perfectly tight. The pieces should be well bolted together at the ends, to prevent any sliding of the parts. In this manner, a beam might be built of any depth that is necessary in the erection of buildings, and by breaking the joints of any length that is likely to be needed in the construction of floors; when built of several pieces, shorter than the bearing, the thickness of the bent pieces may be about one-fiftieth part of the span, and as many of them should be added as will increase the depth to that proposed, unless the whole depth of the curved pieces exceeds half the depth of the girder, and in that case, straight pieces should be added to the underside, so as to make the whole depth of the straight parts exceed the depth of the curved parts.

When pieces cannot be got sufficiently long for the girder, care should be taken to have no joints near the middle of the length in the lower half of the girder.

In the construction of floors, it would be a great advantage to make each girder only half the breadth given by the rule, and to place them only five feet apart; to bridge the upper or floor joists over the girders, to notch the ceiling joists to the under side of them, and to omit the binders. There would be a great increase of strength and stiffness by adopting this method; and, in point of economy, it is decidedly preferable, only it requires a greater depth of flooring.

As the strain is always greatest at the middle of the length of a girder, it would be well to avoid making mortices there, if possible, either for binding joists or any other purpose; and the most straight grained part of the beam should be put to the under side.

Timber girders should not be built into the wall, but an open space should be left around their ends, either by laying a flat stone over them, or by turning an arch to carry the wall above, and the girder laid from 9 to 12 inches into the wall, according to the bearing.

BINDING JOISTS.

The depth of a binding joist is generally determined by the depth of the floor, but this is not always the case. Therefore the rules must be given for two cases.

Case 1.—*To find the depth of a binding joist, the length and thickness being given.*

Rule.—Divide the square of the length in feet, by the thickness; and the cube root of the quotient multiplied by 3·42 for pine, or by 3·53 for oak, will give the depth in inches.

Case 2.—*To find the thickness, when the depth and length are given.*

Rule.—Divide the square of the length in feet, by the cube of the depth in inches, and multiply the quotient by 40 for pine, or by 44 for oak, which will give the thickness in inches.

These rules suppose the distance apart to be 6 feet; if the distance apart be greater or less than 6 feet, the thickness given by the rule must be increased or diminished in proportion. The thickness of the binding joists next the wall may be two thirds of the thickness of the others, but in general they are made the same thickness, and such as are defective are selected for that purpose.

BRIDGING JOISTS.

The rule for bridging joists is the same as that for single joisting. They seldom need be more than 2 inches in thickness, except for ground floors, where they are laid upon sleepers; in which case, the depth may be found to a thickness of 2 inches, and an inch may be added to the thickness on account of the situation; as when proper care is not taken to drain and ventilate the under side of the ground floor, the joists are subject to a very rapid decay. It is a good practice to strew smiths' ashes, or even common ashes, under such floors, to prevent the growth of fungi. The ashes and scoriæ from a foundry, or any ashes that contain much iron, are best. Mr. Batson found this an effectual remedy for the dry rot. He filled a space below the floor, of 2 feet in depth, with anchor-smiths' ashes, and also charred the sleepers.

CEILING JOISTS.

Ceiling joists require to be no thicker than is necessary to nail the laths to; two inches is quite sufficient for that purpose.

To find the depth of a ceiling joist, when the length of bearing and thickness are given.

Rule.—Divide the length in feet by the cube root of the thickness in inches, and multiply the quotient by 0·64 for pine, or 0·67 for oak, which will give the depth in inches required.

If 2 inches be fixed upon for the thickness of ceiling joists, the rule for finding the depth becomes very easy: for half the length in feet gives the depth in inches; that is, if the length be 10 feet, the depth should be 5 inches; if 6 feet, the depth 3 inches; and so on for any other length.

It is better to notch ceiling joists to the under side of the binding joists, and nail them, than to mortice and chase them in; because it requires less labor, it does not weaken the binding joists, and the ceiling stands better.

GENERAL OBSERVATIONS RESPECTING FLOORS.

Girders should never be laid over openings, such as doors and windows, if it be possible to avoid it. In the bearings of floors, the caution of Vitruvius must be attended to; that is, when the joists are supported by external walls of considerable height, the middle part of the joists should never rest upon a partition wall that does not go higher than the floor, otherwise, the unequal settlement of the walls will cause the floor to be unlevel, and most likely fracture the cornices.

Wall plates and templets should be made stronger as the span becomes longer The following proportions may serve for general purposes:

For a 20 feet bearing, wall plates 4½ by 3 inches.
" 30 " " " " 6 by 4 "
" 40 " " " " 7½ by 5 "

Floors should always be kept higher in the middle than at the sides when first framed, as all floors, however well constructed, will settle in some degree. To prevent the unpleasant appearance of open joints in floors, a good method would be, where the case would admit, to tack down the boards for the first year, and in the hottest season of the following year, to nail them down for good; this would cause the floor to remain perfectly close ever afterwards; but if boards be taken up, and the edges shot, they will shrink again, no matter how long they may have lain before.

OF THE CONSTRUCTION OF ROOFS.

In old Gothic buildings, the roof always had a high pitch, its outline formed a striking feature, and in general had a graceful proportion with the magnitude of the building; sometimes, however, it presented a plain surface of too great extent, as the roof of Westminster Hall. Though a high roof is in perfect unison with the aspiring and pyramidal character of Gothic architecture; in the more chaste and classic style of the Greeks, it is a less conspicuous object. Many of the Grecian buildings were never intended to be roofed at all; but where a roof was necessary, it was not attempted to be hidden, but constituted one of the most ornamental parts of the building. Of timber roofs, we have no examples in Grecian buildings; but the beautiful stone roof of the Octagon Tower of Andronicus Cyrrhestes, and that of the Choragic Monument of Lysicrates, are sufficient to show that they were more inclined to ornament than to hide this essential part of a building.

The height of roofs at the present time, is seldom above one-third of the span, and should never be less than one-sixth. The most usual pitch is that when the height is one-fourth of the span, or when the angle with the horizon is 26½ degrees.

The pediments of the Greek temples make an angle of from 12 to 16 degrees with the horizon; the latter corresponds nearly with one-seventh of the span. The pediments of the Roman buildings vary from 23 to 24 degrees: 24 degrees is nearly two-ninths of the span.

The kinds of covering used for timber roofs, are copper, lead, iron, tinned iron, slates of different kinds, tiles, shingles, reeds, straw, and heath. Taking the angle for slates to be 26½ degrees, the following table will show the degree of inclination that may be given for other materials.

Kind of covering.	Inclination to the horizon in degrees.		Height of roof in parts of the span.	Weight upon a square of roofing.
	Deg.	Min.		
Copper, ...	3	50	1/48	100 pounds.
Lead, ...	3	50	1/48	700 "
Slates, large, ...	22	00	1/5	1120 "
Ditto, ordinary, . . .	26	33	1/4	900 "
Ditto, fine, ...	26	33	1/4	500 "
Plain tiles, ...	29	41	2/7	1780 "
Pan tiles, ...	24	00	2/9	650 "
Thatch, of straw, . . .				
Reeds or Heath, . . .	45	00	1/2	

STRENGTH OF MATERIALS.

ADHESION OF NAILS.

Every carpenter is familiar with the use of the nail, and possesses a practical knowledge, more or less accurate, of the force of adhesion of different nails, and in different substances, so as to decide, without difficulty, what number, and of what length, may be sufficient to fasten together substances of various shapes, and subject to various strains. But, interesting as this subject unquestionably is, it has not been till very recently that the necessary experiments have been made to determine, 1st, the adhesive force of different nails when driven into wood of different species, 2d, the actual weight, without impulse, necessary to force a nail a given depth; and 3d, the force required to extract the nail when so driven. The obtaining this useful knowledge was reserved for Mr. B. Bevan, a gentleman well known in the mechanical and scientific world for the accuracy with which his experiments are conducted.

Mr. Bevan observes, that the theoretical investigation points out an equality of resistance to the entrance and extraction of a nail, supposing the thickness to be invariable; but as the general shape of nails is tapering towards the points, the resistance of entrance necessarily becomes greater than that of extraction; in some experiments he found the ratio to be about 6 to 5.

The following Table exhibits the relative adhesion of nails of various kinds, when forced into dry Christiana Deal, at right angles to the grain of the wood:—

Description of nails used.	Number to the pound Avoir.	Inches long.	Inches forced into the Wood.	Pounds requisite to extract.
Fine Springs, . . .	4560	0·44	0·40	22
Ditto, ...	3200	0·53	0·44	37
Threepenny brads, . .	618	1·25	0·50	58
Cast Iron Nails, ...	380	1·00	0·50	72
Sixpenny Nails, . .	73	2·50	1·00	187
Ditto, ...	—	—	1·50	327
Ditto, ...	—	—	2·00	530
Fivepenny Nails. . . .	139	2·00	1·50	320

The percussive force required to drive the common sixpenny nail to the depth of one inch and a half into dry Christiana deal, with a cast iron weight of 6·275 lbs. was four blows or strokes falling freely the space of 12 inches; and the steady pressure to produce the same effect was 400 lbs.

A sixpenny nail driven into dry elm, to the depth of one inch across the grain, required a pressure of 327 pounds to extract it; and the same nail, driven endways, or longitudinally into the same wood, was extracted with a force of 257 pounds.

The same nail driven two inches endways into dry Christiana deal, was drawn by a force of 257 pounds; and to draw out one inch, under like circumstances, took 87 pounds only. The relative adhesion, therefore, in the same wood, when driven transversely and longitudinally, is 100 to 78, or about 4 to 3 in dry elm; and 100 to 46, or about 2 to 1 in deal; and in like circumstances, the relative adhesion to elm and deal is as 2 or 3 to 1.

The progressive depths of a sixpenny nail into dry Christiana deal by simple pressure were as follows:—

One quarter of an inch, a pressure of	24 lbs.
Half an inch, ...	76 —
One inch, ...	235 —
One inch and a half, . . .	400 —
Two inches, ...	610 —

In the above experiments, great care was taken by Mr. Bevan to apply the weight steadily, and towards the conclusion of each experiment, the additions did not exceed 10 lbs. at one time, with a moderate interval between, generally about one minute, sometimes 10 or 20 minutes. In other species of wood, the requisite force to extract the nail was different. Thus, to extract a common sixpenny nail from a depth of one such out of

Dry Oak, required	507 lbs.
Dry Beech, ...	667 —
Green Sycamore, ...	312 —

From these experiments, we may infer that a common sixpenny nail, driven two inches into dry oak, would require a force of more than half a ton to extract it by a steady force.

ADHESION OF SCREWS.

A common screw, of one-fifth of an inch, was found to have an adhesive force of about three times that of a sixpenny nail.

ADHESION OF IRON PINS.

The force necessary to break or tear out a half-inch iron pin, applied in the manner of a pin to a tenon in the mortice, has likewise obtained the attention of the same celebrated experimentalist. The thickness of the board was 0.87 inch, and the distance of the center of the hole from the end of the board 1·05 inch. The force required was 976 pounds.

As the strength of a tenon from the pin-hole may be considered in proportion to the distance from the end, and also as the thickness, we may, for this species of wood, obtain the breaking force in pounds nearly, by multiplying together one thousand times the distance of the hole from the end by the thickness of the tenon in inches.

ADHESION OF GLUE.

Mr. Bevan glued together by the ends two cylinders of dry ash-wood, one-fifth of an inch diameter and about eight inches long; after they had been glued together 24 hours, they required a force of 1260 pounds to separate them; and as the area of the circular ends of the cylinders were 1·76 inches, it follows that the force of 715 pounds would be required to separate one square inch.

It is right to observe, that the glue used in this experiment was newly made, and the season very dry. For in some former experiments on this substance, made in the winter season, and upon some glue which had been frequently made, with occasional additions of glue and water, he obtained a result of 350 to 560 pounds to the square inch.

The present experiment, however, was conducted upon a larger scale, and with greater care in the direction of the resultant force, so that it might be, as near as practicable, in a line passing at right angles through the centers of the surfaces in contact. The pressure was applied gradually, and was sustained two or three minutes before the separation took place.

Upon examining the separated surfaces, the glue appeared to be very thin, and did not entirely cover the wood, so that the actual adhesion of glue must be something greater than 715 pounds to the square inch.

Mr. Bevan also tried the lateral cohesion of fir-wood, from a Scotch fir of his own planting, cut down in the autumn, sawn into boards, and, at the time of experiment, quite dry and seasoned. The force required to separate the wood was 562 pounds to the square inch; consequently, if two pieces of this wood had been well glued together, the wood would have yielded in its substance before the glue.

From a subsequent experiment, made on solid glue, the cohesive force was found to be 4000 pounds per square inch; from which it may be inferred, that the application of this substance as a cement is susceptible of improvement.

STRENGTH OF MATERIALS WHEN EXPOSED TO TRANSVERSE OR CROSS STRAINS.

1.—*To find the Breadth of a uniform Cast Iron Beam, to bear a given weight in the middle.*

Rule.—Multiply the length between the supports in feet, by the weight to be supported in pounds, and divide the product by 850 times the square of the depth in inches; the quotient will be the required breadth in inches.

2.—*To find the Depth of a uniform Cast Iron Beam, to bear a given weight in the middle.*

Rule 1.—Multiply the length of bearing in feet, by the weight to be supported in pounds, and divide this product by 850 times the breadth in inches; and the square root of the quotient will be the required depth in inches.

When no particular breadth or depth is determined by the nature of the situation for which the beam is intended, it will be found sometimes convenient to assign some proportion; as, for example, let the breadth be the nth part of the depth, n representing any number at will. Then the rule will be as follows:

Rule 2.—Multiply n times the length in feet, by the weight in pounds; divide this product by 850, and the cube root of the quotient will be the depth required: the breadth will be the nth part of the depth.

Note.—It may be remarked here, that the rules are the same for inclined as for horizontal beams, when the horizontal distance between the supports is taken for the length of bearing.

Example.—In a situation where the flexure of a beam is not a material defect, it is required to support a load which cannot exceed 33600 pounds, or 15 tons, in the middle of a cast iron beam, the distance of the supports being 20 feet; and making the breadth a fourth part of the depth. In this case,

$$n = 4 \text{ and } \frac{4 \times 20 \times 33600}{850} = 3162{\cdot}35$$

The cube root of 3162·35 is nearly 14·68 inches, the depth required; the breadth is

$$14{\cdot}68 \div 4 = 3{\cdot}87 \text{ inches.}$$

In practice, therefore, whole numbers should be used, and the beam be made 15 inches in depth, and 4 inches in breadth.

3.—*To find the Breadth of a uniform Cast Iron Beam, when the load is not in the middle between the supports.*

Rule.—Multiply the distance between the load and the nearest support in feet, by the distance between the load and the furthest support in feet, and four times this product, divided by the whole length between the supports, will give the effective leverage of the load in feet; this quotient being used instead of the length, in any of the foregoing Rules, will give the breadth and depth.

Example.—Instead of placing 15 tons in the middle of a beam, as in the last Example, let it be placed at 5 feet from one end, and 15 feet from the other.

$$\frac{5 \times 15 \times 4}{20} = 15,$$

which is the number to be employed instead of the whole length, as in the last example; that is

$$\frac{4 \times 15 \times 33600}{850} = 2372 \text{ nearly.}$$

And the cube root of 2372 is nearly 13·34 inches, the depth of the required beam; and

$$13.34 \div 4 = 3{\cdot}33 \text{ inches}$$

gives the breadth; or the beam is nearly 13½ inches by 3½ inches. In the foregoing Example it was 15 inches by 4 inches.

4.—*To substitute Beams of Wrought Iron, of Oak, or of Yellow Fir, instead of having Beams of Cast Iron, in either of the foregoing problems.*

Rule.—Instead of using 850 as a divisor, when the beam is of wrought iron, employ 952; when it is of oak, 212; and when it is of yellow fir, 255.

5.—*To find the Breadth of a uniform Cast Iron Beam, when the load is distributed over the length of a Beam, supported at both ends.*

Rule.—When the load is uniformly distributed over the length of a beam, it supports double the weight that it would do if the load were laid on the middle; therefore the divisor, in the preceding examples, is changed from 850 to 1700.

Example.—In a situation where an arch cannot be used for want of abutments, it is necessary to leave an opening 15 feet wide, in an 18-inch brick wall; required the depth of two cast iron beams to support the walls over the openings, each beam to be 2 inches thick, and the height of the wall intended to rest on the beam being 30 feet?

The wall contains

$$30 \times 15 \times 1\tfrac{1}{2} = 675 \text{ cubic feet;}$$

and as a cubic foot of brickwork weighs about 100 lbs., the weight of the wall will be about 67500 lbs.; and half this weight, or 33750 lbs., will be the load upon one of the beams. Since the breadth is supposed to be given, the depth will be found by problem 2, if 1700 be used as the constant divisor; thus,

$$\frac{15 \times 33750}{1700 \times 2} = 149 \text{ nearly.}$$

The square root of 149 is 12¼ nearly; therefore each beam should be 12¼ inches deep, and 2 inches in thickness. This operation gives the actual strength necessary to support the wall; but double the weight is usually taken in practice to allow for accidents. In this manner the strength for brestsummers, lintels, and the like, may be determined.

6.—*To find the Dimensions of a Cast or Wrought Iron Beam, when it is to be fixed at one end and loaded at the other; or when it is to be supported at the middle, and loaded at both ends.*

Rule.—When the beam is fixed at one end, take the length of the projection of the beam, for the length, and apply the rules in Problems 1 and 2, using the 212 instead of 850 as a divisor. By the same rule, the proportions for the arms of a balance may be found.

When the beam is of wrought iron, employ 238 as a divisor

Example.—Let the length of the arms, from the center of suspension to the center of motion, be 1½ feet, and the extreme weight 3 cwt., or 336 pounds, and let the thickness be one-tenth part of the depth; then by the rule

$$\frac{10 \times 1{\cdot}5 \times 336}{212} = 24 \text{ nearly.}$$

The cube root of 24 is 2.88 inches, which will be the depth of the beam at the center; and the breadth will be 0·288 inch.

When the weight is uniformly distributed over the length of the beam, employ 425 instead of 850, as a divisor.

7.—*To find the Dimensions of the Teeth of Wheels.*

Rule.—Divide the stress at the pitch-circle in pounds by 1500, and the square root of the product is the thickness of the teeth in inches.

Example.—Let the greatest power acting at the pitch-circle of a wheel be 6000 pounds; then

$$6000 \div 1500 = 4,$$

and the square root of 4 is 2 inches, the thickness required.

The breadth of teeth should be proportional to the stress upon them, and this stress should not exceed 400 pounds for each inch in breadth.

The length of teeth ought not to exceed the thickness, but the strength is not affected by the less or greater length of the teeth.

8.—*To find the Diameter of a Solid Cylinder to support a given weight in the middle.*

Rule.—Multiply the weight in pounds by the length in feet; divide this product by 500, and the cube root of the quotient will be the diameter in inches.

9.—*To find the Diameter of a Solid Cylinder to support a weight between the middle and the end.*

Rule.—Multiply the short end by the long end; then multiply that product by four times the weight in pounds. Divide this product by 500 times the length in feet, and the cube root of the quotient will be the diameter in inches.

10.—*To find the Diameter of a Solid Cylinder when the load is uniformly distributed over the length.*

Rule.—Multiply the length in feet by the weight in pounds, and one-tenth of the cube root of the product will be the diameter in inches.

11.—*To find the Diameter of a Solid Cylinder when fixed at one end, and having the load applied at the other.*

Rule.—Multiply the length of projection in feet by the weight in pounds, and the 5th part of the cube root of this product will be the diameter in inches.

STRENGTH OF MATERIALS WHEN EXPOSED TO A LATERAL STRAIN.

The rules given in Problems 8, 9, 10, and 11, can be here applied.

Example.—What is the diameter of the journal of a water-wheel shaft, 13 feet long, the weight of the wheel being 15 tons or 33600 lbs.

When the weight is in the middle $\frac{33600 \times 13}{500} = 873$, and the cube root of 873 is 9½ inches, the diameter of the required journal.

When the weight is equally distributed, $33600 \times 13 = 436800$, and the cube root of 436800 divided by 10 will give 7·65 inches, the diameter required.

WEIGHTS REQUIRED TO CRUSH ONE CUBIC INCH OF SEVERAL MATERIALS USED IN THE CONSTRUCTION OF BUILDINGS.

METALS.		WOODS.	
Cast Iron,	116,700	Elm,	1,284
Brass,	154,784	American Pine,	1,606
Copper, Cast,	116,102	White Deal,	1,928
Lead, Cast,	8,042	White Oak,	3,240
		English Oak,	3,860
STONES.			
Freestone,	18,000	Brick, hard,	1,754
Limestone, Black,	19,450	Brick, soft,	1,224
Granite, Blue,	20,890	Chalk,	1,040

OF POSTS.

According to the experiments of Rondelet, when the height of a square post is less than about 7 or 8 times the side of its base, it cannot be bent by any pressure less than that which would crush it. The internal mechanism of the resisting forces, when timber yields by crushing, is not exactly understood. In timber the resistance to crushing is less than the cohesive force. The resistance of timber to crushing appears to increase in a higher ratio than that of the area of its section.

The load a piece of timber will bear, when pressed in the direction of its length, without risk of being crushed, may be found by the following rule;

Multiply the area of the piece of timber, in inches, by the weight that is capable of crushing a square inch of the same kind of wood, then, one-fourth of the product will give the load in pounds that the piece would bear with safety.

If the area that would support a given weight be required, divide 4 times the weight by the number of pounds that would crush a square inch, and the quotient is the area in inches.

The length should never exceed 10 times the side of the section to give the above results; for when the length is greater than about 10 times the thickness, the piece will bend before it crushes.

WEIGHT OR FORCE REQUIRED TO TEAR ASUNDER ONE SQUARE INCH OF THE DIFFERENT MATERIALS USED IN THE CONSTRUCTION OF BUILDINGS.

WOODS AND METALS.

Oak, American, . .	17,300	Swedish Iron, . .	78,850
Oak, English, . . .	19,800	English Iron, . . .	55,772
Beach, . . .	17,700	French Iron, . .	61,041
Ash, ...	16,700	Russian Iron, . . .	59,472
Elm, ...	13,489	Cast Iron, . . .	42,000
Walnut, ...	8,130	Steel, soft, . . .	120,000
Norway Pine, . .	14,300	Ivory, . . .	16,000
Georgia Pine, . . .	7,818	Marble, ...	8,700
White Pine, . .	8,800	Whalebone, . .	7,600
Iron Wire, . . .	113,077		

To find the strength of Cohesion.

Rule.—Multiply area of section in inches by the weight required to tear one inch asunder, and the product is the strength in pounds.

WEIGHT IN POUNDS OF CUBIC FOOT OF WOOD AND STONE.

WOOD.		STONE.	
Apple-tree,	49.6	Flint,	163.2
Ash,	52.9	Blue Granite,	164.1
Birch,	33.2	Lime Stone,	199.
American Cedar,	35.1	Grindstone,	134.
Elm,	42.	Slate Stone,	167.
White Pine,	35.6	Marble,	170.
Yellow Pine,	41.1	Free Stone,	150.
Mahogany,	66.5	African Marble,	169.2
Maple,	47.	Egyptian Marble,	166.8
Mulberry,	56.1	Italian Marble,	166.1
Oak,	58.74	Roman Marble,	172.2
Live Oak,	70.		

OTHER SUBSTANCES.

Cast Iron,	450.55	Air,	.07529
Wrought Iron,	486.65	Steam,	.03689
Steel,	489.8	Loose Earth or Sand,	95.
Copper,	555.	Common Soil,	124.
Lead,	708.75	Strong Soil,	127.
Brass,	537.75	Clay,	135.
Tin,	456.	Clay and Stones,	160.
Salt Water, (Sea),	64.3	Cork,	15.
Fresh Water,	62.5	Tallow,	59.
		Brick,	125

WEIGHT OF ROUND AND SQUARE IRON,

FROM ONE FOURTH TO FIVE INCHES DIAMETER AND ONE FOOT LONG.

ROUND IRON.		SQUARE IRON.	
Size of Iron.	1 Foot.	Size of Iron.	1 Foot.
Inches.	*lbs.*	*Inches.*	*lbs.*
One Fourth,	·163	One Fourth,	·208
Three Eighths,	·368	Three Eighths,	·468
One Half,	·654	One Half,	·833
Five Eighths,	1·02	Five Eighths,	1·3
Three Fourths,	1·47	Three Fourths,	1·87
Seven Eighths,	2·	Seven Eighths,	2·55
One Inch,	2·61	One Inch,	3·33
One Eighth,	3·31	One Eighth,	4·21
One Fourth,	4·09	One Fourth,	5·2
Three Eighths,	4·94	Three Eighths,	6·3
One Half,	5·89	One Half,	7·5
Five Eighths,	6·91	Five Eighths,	8·8
Three Fourths,	8·01	Three Fourths,	10·2
Seven Eighths.	9·2	Seven Eighths,	11·71
Two Inches.	10·47	Two Inches.	13·33
One Eighth,	11·82	One Eighth,	15·05
One Fourth,	13·25	One Fourth,	16·87
Three Eighths,	14·76	Three Eighths,	18·8
One Half,	16·36	One Half,	20·8
Five Eighths,	18·03	Five Eighths,	22·96
Three Fourths,	19·79	Three Fourths,	25·2
Seven Eighths,	21·63	Seven Eighths,	27·55
Three Inches.	23·56	Three Inches,	30·
One Eighth,	25·56	One Eighth,	32·55
One Fourth,	27·65	One Fourth,	35·2
Three Eighths,	29·82	Three Eighths,	37·96
One Half,	32·07	One Half,	40·8
Five Eighths,	34·4	Five Eighths.	43·8
Three Fourths,	36·81	Three Fourths,	46·87
Seven Eighths,	39·31	Seven Eighths,	50·05
Four Inches,	41·88	Four Inches,	53·33
One Eighth,	44·54	One Eighth,	56·71
One Fourth,	47·28	One Fourth,	60·02
Three Eighths,	50·11	Three Eighths,	63·8
One Half,	53·01	One Half,	67·5
Five Eighths,	56·	Five Eighths,	71·3
Three Fourths,	59·06	Three Fourths,	75·2
Seven Eighths,	62·21	Seven Eighths,	79·21
Five Inches.	65·45	Five Inches,	83·33

WEIGHT OF FLAT LEAD, PER SUPERFICIAL FOOT,

ONE SIXTEENTH TO ONE INCH THICK.

Size.	1-16t.	⅛	3-16t.	¼	5-16t.	⅜	7-16t.	½
Weight.	*lbs.* 3·7.	*lbs.* 7·4	*lbs.* 11·1	*lbs.* 14·81	*lbs.* 18·51	*lbs.* 22·22	*lbs.* 25·92	*lbs.* 29·62
Size.	9-16t.	⅝	11-16t.	¾	13-16t.	⅞	15-16t.	1 in.
Weight.	*lbs.* 33·33	*lbs.* 37·03	*lbs.* 40·73	*lbs.* 44·44	*lbs.* 48·14	*lbs.* 51·86	*lbs.* 55·55	*lbs.* 59·25

WEIGHT OF TIN,

[AS MANUFACTURED IN ENGLAND.]

	Brand Mark.	No. Sheets per box.	Length and Breadth.	Weight per box.		
				cwt.	*qrs.*	*lbs.*
One Box of	I C or I com.. .	225	13¾ In by 10 In.	1	0	0
	2 C, . . .	225	13¼ - - 9¾	0	3	21
	3 C, . . .	225	12¾ - - 9½	0	3	14
	H C, . . .	225	13¾ - - 10	1	0	7
	Hx, . . .	225	13¾ - - 10	1	1	7
	Ix, . . .	225	13¾ - - 10	1	1	0
	2x, ...	225	13¼ - - 9¾	1	0	21
	3x, . . .	225	12¾ - - 9½	1	0	14
	Ixx, . . .	225	13¾ - - 10	1	1	21
	Ixxx, . . .	225	13¾ - - 10	1	2	14
	Ixxxx, . . .	225	13¾ - - 10	1	3	7
	D C, . . .	100	16¾ - - 12½	0	3	21
	Dx, . . .	100	16¾ - - 12½	1	0	14
	Dxx, . . .	100	16¾ - - 12½	1	1	7
	Dxxx, . . .	100	16¾ - - 12½	1	2	0
	Dxxxx, . .	100	16¾ - - 12½	1	2	21
	SDC, . . .	200	15 - - 11	1	2	0
	SDx, . . .	200	15 - - 11	1	2	21
	SDxx, . . .	200	15 - - 11	1	3	14
	SDxxx, . .	200	15 - - 11	2	0	7
	SDxxxx, . .	200	15 - - 11	2	1	0
	Wasters or Wc&xmixd, .	225	13¾ - - 10	1	0	14
	TT, . . .	450	13¾ - - 10	1	0	0
	XTT, . . .	450	13¾ - - 10	1	0	14

5,280 feet make one mile.

Steel wire will sustain 39,000 feet of its own length.

600 cubic feet of hay, if well packed, will weigh one ton; but in deep bays, 500, and even 400, in some cases, will weigh as much.

It requires eleven months to season, in the open air, a piece of timber 10 feet long and 6 × 6 inches in thickness; fifteen months for a piece of the same length and 8 × 8 inches in thickness; twenty-two months for the same 12 × 12 inches; twenty-five months for the same 14 × 14 inches; twenty-nine months for the same 16 × 16 inches; thirty-two months for the same 18 × 18 inches; and thirty-six months for the same 20 × 20 inches.

FEDERAL MONEY.

		dwt.	gr.	
10 mills = 1 cent.	The cent weighs	6	23	copper.
10 cents = 1 dime.	The dollar "	17	1¾	silver.
10 dimes = 1 dollar.	The eagle "	11	4⅔	gold.
10 dollars = 1 eagle.				

The standard for Federal money of gold and silver is 11 parts fine, and 1 part alloy.

The full weight and value of the English gold and silver coin is as follows:

Gold.	Value.			Weight.		Silver.	Value.		Weight.	
	£.	s.	d.	dwt.	gr.		s.	d.	dwt.	gr.
A Guinea,	1	1	0	5	19½	A Crown,	5	0	19	8½
Half Guinea,	0	10	6	2	16¾	Half Crown,	2	6	9	16½
Seven Shillings,	0	7	0	1	19⅙	Shilling,	1	0	3	21
Quarter Guinea,	0	5	3	1	8¼	Sixpence,	0	6	1	22½

The usual value of gold is nearly £4 an ounce, or 2d. a grain; that of silver is nearly 5s. 3d. an ounce. The value of any quantity of gold is to the value of the same weight of standard silver, nearly as 15 to 1, or more nearly, as 15¼ to 1.

Pure gold, free from mixture with other metals, usually called fine gold, is of so pure a nature, that it will endure the fire without wasting, though it be kept continually melted. But silver, not having the purity of gold, will not, like it, endure the fire. Fine silver will, however, waste but little, if suffered to remain a moderate time in the fire.

WEIGHT OF ROUND AND SQUARE IRON,

FROM ONE FOURTH TO FIVE INCHES DIAMETER AND ONE FOOT LONG.

ROUND IRON.		SQUARE IRON.	
Size of Iron.	1 Foot.	Size of Iron.	1 Foot.
Inches.	*lbs.*	*Inches.*	*lbs.*
One Fourth,	·163	One Fourth,	·208
Three Eighths,	·368	Three Eighths,	·468
One Half,	·654	One Half,	·833
Five Eighths,	1·02	Five Eighths,	1·3
Three Fourths,	1·47	Three Fourths,	1·87
Seven Eighths,	2·	Seven Eighths,	2·55
One Inch,	2·61	One Inch,	3·33
One Eighth,	3·31	One Eighth,	4·21
One Fourth,	4·09	One Fourth,	5·2
Three Eighths,	4·94	Three Eighths,	6·3
One Half,	5·89	One Half,	7·5
Five Eighths,	6·91	Five Eighths,	8·8
Three Fourths,	8·01	Three Fourths,	10·2
Seven Eighths.	9·2	Seven Eighths,	11·71
Two Inches.	10·47	Two Inches.	13·33
One Eighth,	11·82	One Eighth,	15·05
One Fourth,	13·25	One Fourth,	16·87
Three Eighths,	14·76	Three Eighths,	18·8
One Half,	16·36	One Half,	20·8
Five Eighths,	18·03	Five Eighths,	22·96
Three Fourths,	19·79	Three Fourths,	25·2
Seven Eighths,	21·63	Seven Eighths,	27·55
Three Inches.	23·56	Three Inches,	30·
One Eighth,	25·56	One Eighth,	32·55
One Fourth,	27·65	One Fourth,	35·2
Three Eighths,	29·82	Three Eighths,	37·96
One Half,	32·07	One Half,	40·8
Five Eighths,	34·4	Five Eighths.	43·8
Three Fourths,	36·81	Three Fourths,	46·87
Seven Eighths,	39·31	Seven Eighths,	50·05
Four Inches,	41·88	Four Inches,	53·33
One Eighth,	44·54	One Eighth,	56·71
One Fourth,	47·28	One Fourth,	60·02
Three Eighths,	50·11	Three Eighths,	63·8
One Half,	53·01	One Half,	67·5
Five Eighths,	56·	Five Eighths,	71·3
Three Fourths,	59·06	Three Fourths,	75·2
Seven Eighths,	62·21	Seven Eighths,	79·21
Five Inches.	65·45	Five Inches,	83·33